Volume XIX, 36 – No. 1, 2017

DETERMINING REFERENCE VALUES OF LQI OUTPUT VARIABLES FOR A SMALL SCALE TURBOJET WITH VARIABLE EXHAUST NOZZLE

André Ferenc GOGOLA*
Budapest University of Technology And Economics
**Corresponding author.* E-mail: gogola.andre@gmail.com

Summary. Aero-engines are complex, non-linear systems from the perspective of engine control, thus designing engine control that is efficient throughout the operating envelope is also a complex task. Current aero-engine control researches, whose objective is to cover the operating range of the engine are mentioned to receive an insight from the engine control development trends. The aim of this work is to initiate the design process of a gain scheduling linear quadratic integral (LQI) control of a single spool variable exhaust nozzle small scale turbojet engine by applying an algorithm, that converts the thrust lever angle positions into rotor speed and engine pressure ratio reference values for the controller, while the thrust force of the engine changes linearly with the thrust lever angle position. This work presents the measurement that provides the dataset for the algorithm, and uses the linearized state space model of the turbojet to determine the variables for which the reference values must be obtained. Then the calculation process for the determination of reference points based on the measurement data is detailed. Finally, the results of the applied process for the investigated small scale turbojet engine are indicated.

Keywords: reference values; LQI control, turbojet engine, linearized model

1. INTRODUCTION

Gas turbines are commonly used in various segments of the industry for power generation. Besides being used in power-plants for electrical and heat energy production from the hundreds kW power range to the hundreds of MW power range [11],[12], gas turbines are also efficiently used in the transportation industry. Aviation industry is the most significant area, where numerous types and sizes of gas turbines exist. Aviation gas turbines are primarily used for aircraft propulsion with a wide operating envelope; but auxiliary power units on large aircraft are also used, which are almost without exception constant speed gas turbines, and they provide electrical power, pressurized air and occasionally hydraulic power for the systems of the aircraft. Gas turbines can provide efficient propulsive power for transport aircraft in the form of high bypass ratio turbofans with mass flows in excess of 1000 kg/s [13], high thrust at a low bypass ratio for fighter aircraft with afterburning capabilities [1],[8]; it can also be integrated with a reduction gearbox to drive a propeller [2], and can also be fitted on unmanned air vehicles (UAVs) in the form of micro gas turbines [17].

Aviation gas turbines used for propulsion have a wide range of operation, where efficient control of the engine must be ensured. The gas turbine engine is a complex time-varying multivariable non-linear system from the controlling perspective [18], which makes the development of a fast and efficient controller covering the whole operating range a challenging task, as a single linear controller is efficient only in the vicinity of one specified working point of the gas turbine. Recent studies investigated developing gain scheduling controllers [3],[19], where the gas turbine characteristics are linearized around chosen design points. A linear controller is designed on each point, and by using an interpolating algorithm, a nonlinear control law can be designed throughout the gas turbine operating range [18]. Gain scheduling based on a Linear Parameter Varying (LPV) system is different from traditional gain scheduling method, because it designs a controller directly for the whole operating range [18]. LPV with gain scheduling is widely applied in aero-engines [20], however to achieve superior description of nonlinear dynamics over the LPV controller, researches were executed on developing switched LPV controllers [20],[21]. A different method for obtaining non-linear engine control is by applying Equilibrium Manifold Expansion (EME) model, where mapping around the equilibrium points connect the equilibrium point to the appropriate linear model [4]. Model Predictive Control (MPC) has also been recently investigated for numerous gas turbine control applications [[5], [6], [22], [23]], real-time implementation is also researched, however the computational capacity of real time MPC models is significant. Usually a model reduction is required for real-time implementation. Yet, the application of MPC is limited in aero-engines [24], however it can be found in industrial applications [6]. To achieve efficient control over wider operating range, adaptive MPC is investigated by Du et al [5].

In the second part of this work, the measurement for the calculations process is presented, followed by the mathematical model of the turbojet and the explanation of the reason for calculating reference values. The third chapter will detail the calculation process, while the fourth chapter indicates the results of the application of the process.

2. MEASUREMENT DATA, MODEL LQI CONTROL PRELIMINARIES

2.1 The investigated turbojet and measurement data

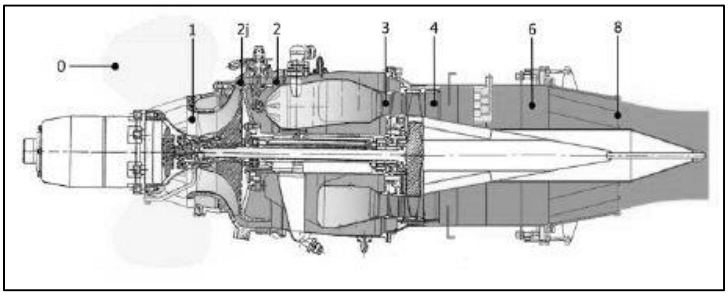

Figure 1. Aerodynamic stations of the investigated turbojet engine TKT-1 [7]

The turbojet engine investigated in this work is the TKT-1 single spool turbojet engine with variable convergent exhaust nozzle; the modified version of the auxiliary power unit TS-21. The cross section view with the aerodynamic stations of the jet engine is demonstrated in Figure 1. The dataset for the calculation process is obtained by quasi-stationary measurements on a test stand with the ambient and boundary conditions tabulated in Tab. 1.

Table 1. Results of calculations process for ten reference points

Boundary Condition	Exhaust nozzle area A_8 [cm^2]	Rotor speed range [100/min]		Ambient conditions	
		idle	full thrust	p_0^* [kPa]	T_0^* [°C]
exhaust nozzle open	75	303,19	450.17	98.55556	15.9
exhaust nozzle closed	150	209.69	427.64		

The purpose of the measurement was to map the operating envelope of the turbojet and to provide a data set for the calculation of reference values. The measurements mapped the operating envelope of the engine under two conditions: variable exhaust nozzle closed, variable exhaust nozzle open. The rotor speed range against time is indicated in Figure 2, the rotor speed in both conditions varied from engine idle to full thrust setting. As it can be seen on Figure 2, the engine was slowly accelerated from idle at 1 minute and achieved full thrust at 2 minute 1 seconds for the exhaust nozzle closed position and the slope abruptly started to converge to horizontal for the exhaust nozzle open position. The acceleration time of approximately one minute is deemed by the author to be gentle enough to consider the measurement quasi stationary.

The investigated gas turbine engine incorporates a minimum fuel supply protection to avoid flame out, and also a maximum fuel pressure protection, to avoid over speed of the rotor; thus due to these two protections, the fuel supply range during both measurements were the same. Figure 2 indicates that the engine idle rotor speed is approximately 10000 rpm lower with the exhaust nozzle in the closed position, which is due to the increased load on the engine by the reduced outflow area.

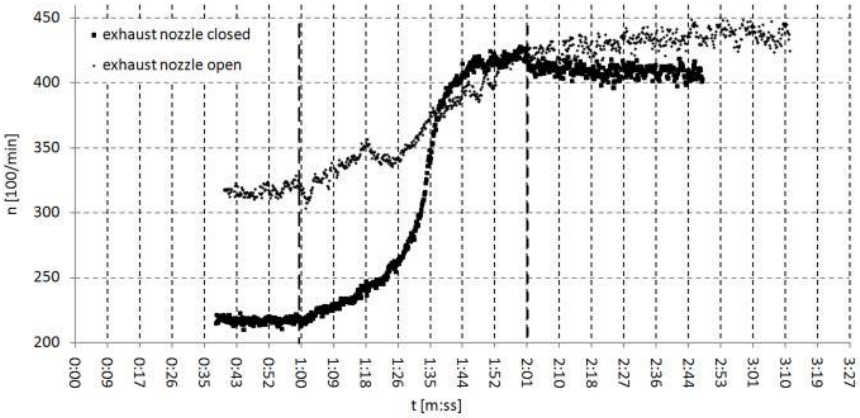

Figure 2. Rotor speed against time during the measurements

The data directly obtained from the measurement throughout the operating envelope – which will be further utilized in this work – are the total inlet pressure p_1^*, the total temperature and pressure at the turbine outlet T_4^* and p_4^*. The rotor speed n and mass flow at the variable exhaust nozzle outlet $\dot{m}_8$ were determined from the measurement data by using thermo-dynamical equations, while the engine pressure ratio (EPR) was determined from the measurement data based on its definition. The measured variables for both conditions will be indicated later on the figures prepared to determine the reference values.

2.2. Mathematical model of the investigated turbojet engine

The mathematical model of the investigated turbojet engine is linearized by first order Taylor series linearization method to establish an accurate model in the vicinity of the chosen working point with small computation penalty. The linearized equations are represented in a state space model, written in the deviational form [7]:

$$\dot{\tilde{x}}(t)=\boldsymbol{A}\tilde{x}(t)+\boldsymbol{B}\tilde{u}(t) \qquad (1)$$
$$\tilde{y}(t)=\boldsymbol{C}\tilde{x}(t)+\boldsymbol{D}\tilde{u}(t)$$

In equation (1), $\tilde{x}$, $\tilde{u}$ and $\tilde{y}$ stand for the transformed state, the input and output vectors, **A**, **B**, **C** and **D** represent the system, input, output and feedforward matrices. The matrices of the state space equations are identified by measurements in [25]. The state variable of the turbojet engine stand of five elements [14], that are the rotor speed n, total pressure and temperature at the turbine inlet p_3^* and T_3^*, exhaust nozzle inlet total temperature p_6^* and T_6^* (2):

$$x=[n\ \ p_3^*\ \ T_3^*\ \ p_6^*\ \ T_6^*]^T \qquad (2)$$

There are two possible parameters of the engine that can be changed during the operation, thus the input vector consists of these two elements, such as the fuel mass flow rate $\dot{m}_{fuel}$ and the outlet area of the exhaust nozzle A_8 as described in (3):

$$u=[\dot{m}_{fuel}\ A_8]^T \tag{3}$$

The primary purpose of the engine control is to maintain constant thrust output at a given position of the throttle lever. The system consists of two output variables; this means that there are two output parameters which indicate the thrust force generated by the engine. Conventionally, the two variables selected are the engine rotor speed and engine pressure ratio (EPR), based on the reports of [26]. The output vector is represented in (4):

$$y=[n\ EPR]^T \tag{4}$$

EPR can be defined as the ratio of the exhaust nozzle inlet total pressure p_8^* and the compressor inlet total pressure p_1^*:

$$EPR=\frac{p_8^*}{p_1^*} \tag{5}$$

2.3. Reference values for the turbojet LQI controller

Numerous instabilities may arise in a jet engine around a working point [9] due to real environment disturbances which are not modelled. Blade flutter, rotating stall, and thermo-acoustic instabilities – if uncontrolled – may couple with resonant modes and lead to fluctuations of the output variables and even to engine damage [27]. To stabilize the output variables and the thrust force, an LQI optimal controller was developed for the investigated turbojet engine in [28]. The selected LQI technology is able to suppress disturbances at the design point and maintain reference values.

The aim of the present work is to initiate the expansion of the LQI controller from the chosen working point to the operating envelope of the investigated turbojet engine by describing the process for determining the reference values of the output variables. By this method, also a relationship can be created between the thrust lever angle and the thrust force produced, which is advantageous from the operational perspective. The established reference values will be utilized to design a gain scheduling controller based on LQI control which covers the engine operating range.

3. ESTABLISHING OPERATING ENVELOPE AND CALCULATION PROCESS

3.1. Establishing the operating envelope for the reference values

Prior to the establishment of the reference values, the considered operating envelope of the engine needs to be nominated carefully. As it was visible on Fig. 2Figure 2, the idle speed of the engine with closed exhaust nozzle is approximately 10000 rpm lower – around 21000 rpm –, than the idle speed with variable exhaust nozzle open. The idle speed for the engine is chosen with the variable exhaust nozzle open condition, as it has two advantages over the other closed exhaust nozzle conditions. Figure 4 a) shows, that lower gas temperatures can be maintained, which saves engine life. On the other hand, engine acceleration from the exhaust nozzle open idle is faster, than for the closed exhaust nozzle case. It means that a more responsive engine could be achieved.

Due to the high thrust force value indicated by Figure 3, the full thrust setting of the engine is chosen at 45000 rpm with the exhaust nozzle closed position. Although measurement data with close exhaust nozzle is available only up to 42764 rpm, the engine still operates in the safe range at 45000 rpm. On Figure 3 and Figure 4, this is the reason why the reference value curves exceed the measurement data. At the current control setting of the engine, the 45000 rpm would be achieved by supplying more fuel into the combustion chamber, while the exhaust nozzle is in the fully closed position.

3.2. Calculation process for determining the reference points

It is essential for the aviation turbojet operator to establish a linear relationship between the throttle lever angle and the thrust produced by the engine. This enables the operator easy and quick thrust selection with the throttle lever. The calculation process for the reference values for the gain scheduling LQI control is based on this requirement. The thrust force data obtained from the measurement throughout the operating envelope with closed and open variable exhaust nozzle was evaluated and used to create a linear relationship between the idle throttle lever position (with the variable nozzle fully open) and the maximum thrust position of the throttle lever (with the variable nozzle fully closed). These two extremities of the operating envelope represent the engine idle and full thrust position. Linear interpolation technique is used to calculate the reference thrust values at regular throttle lever angle (TLA) intervals (6):

$$F_t=\frac{F_{t,max}-F_{t,min}}{TLA_{max}-TLA_{min}}TLA_{ref}+F_{t,min} \tag{6}$$

The established thrust reference values and the data obtained from the measurements are demonstrated in Figure 3.

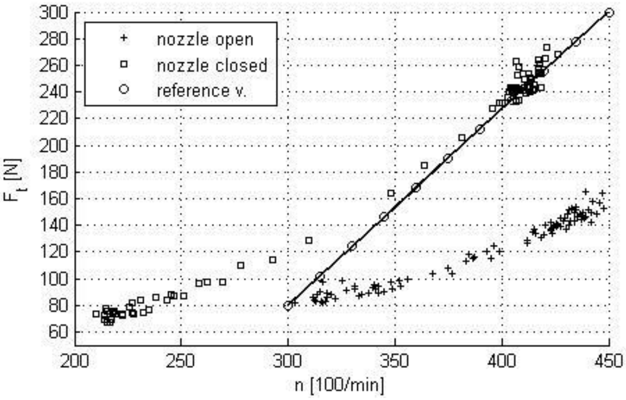

Figure 3. Established reference thrust values

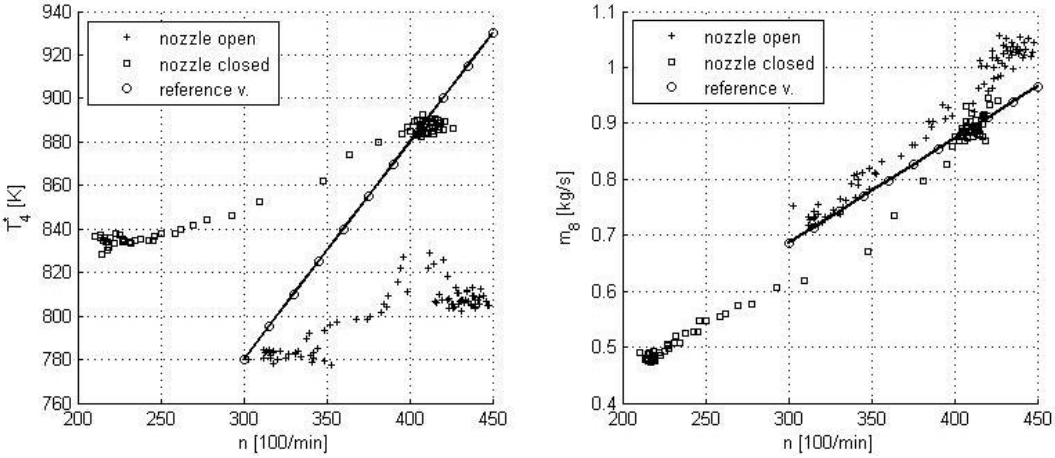

Figure 4. Total turbine outlet temperature a), mass flow against rotor speed b)

It is not possible to directly measure thrust force values of the jet engine when it is installed on the aircraft [10], thus the calculated reference thrust values need to be converted to EPR values. A linear relationship is established based on the measurement from engine idle to full thrust force between the rotor speed and the mass flow at the exhaust nozzle outlet, and between rotor speed and the total turbine outlet temperature as well (7), (8):

$$\dot{m}_8=0{,}00187n+0{,}1245 \tag{7}$$

$$T_4^*=n+480 \tag{8}$$

The exhaust nozzle of the investigated turbojet engine is unchoked, which means that the jet efflux expands to ambient pressure. Consequently, the speed of the outflow is proportional solely to the thrust force and the mass flow over the variable exhaust nozzle, which was already approximated in (9). The known speed of the flow at the outlet of the exhaust nozzle makes it possible to determine the total pressure at this station with an approach using the critical speed and gas dynamic equations [15]:

$$c_8=\frac{F_t}{\dot{m}_8} \tag{9}$$

$$c_{cr}=\sqrt{\frac{2\gamma}{\gamma+1}R\,T_4^*};\qquad \lambda=\frac{c_8}{c_{cr}} \tag{10}$$

$$p_8^*=p_0\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{11}$$

In equations (10) and (11) the change of the specific heat and thus the specific heat ratio (γ) with temperature is approximated with a polynomial based on [16]. Finally, the reference point of the EPR at each chosen thrust and rotor speed value can be determined by equation (5).

4. RESULTS

The previously described calculation process was executed for ten reference points. Tab. 2 presents the results for the steps of the process and the determined reference values for the output variables. The received reference points will be utilized to design a gain scheduling LQI control covering the whole operating envelope for the investigated turbojet engine. As the table indicates, the main achievement of the calculations process is, that based on only one operating variable – the throttle lever angle – the desired reference output of a multivariable system is determined, which ensures a linearly changing thrust force value with the throttle lever angle position.

The results show that a nearly linear relationship exists between the engine pressure ratio and the thrust force, and between engine pressure ratio and the rotor speed as well. The non-linearity exists due to the non-linear variation of the exhaust nozzle speed with the increasing rotor speed, and to the non-linear gas dynamic functions which describe the exhaust nozzle total pressure, and the temperature dependency of the specific heat ratio. The engine pressure ratio reference values were determined based on a constant total inlet pressure throughout the measurement. Corresponding to the basic measurement, it is visible on Figure 5 that to increase the thrust force, both the engine EPR and rotor speed must increase. Rotor speed is mainly controlled by the mass flow of the fuel into the combustion chamber, while changing the exhaust nozzle area with the variable exhaust nozzle, makes possible adjusting the engine pressure ratio.

Table 2. Results of calculations process for ten reference points

TLA [%]	**F_t [N]**	**n [100/min]**	**EPR**	$\dot{m}_8$ [kg/s]	c_8 [m/s]	T_4^*[K]	p_8^* [bar]
0	80,00	300,00	1,0305	0,69	115,76	780,00	1,0442
10	102,00	314,50	1,0452	0,72	141,65	794,50	1,0591
20	124,00	329,00	1,0613	0,75	165,53	809,00	1,0753
30	146,00	343,50	1,0781	0,78	187,64	823,50	1,0924

40	168,00	358,00	1,0954	0,81	208,15	838,00	1,1099
50	190,00	372,50	1,1129	0,84	227,25	852,50	1,1277
60	212,00	387,00	1,1304	0,87	245,06	867,00	1,1454
70	234,00	401,50	1,1477	0,89	261,72	881,50	1,1629
80	256,00	416,00	1,1647	0,92	277,33	896,00	1,1802
90	278,00	430,50	1,1813	0,95	291,99	910,50	1,1970
100	300,00	445,00	1,1974	0,98	305,78	925,00	1,2133

The relationship between the thrust force and the output variables is demonstrated in Fig.5.

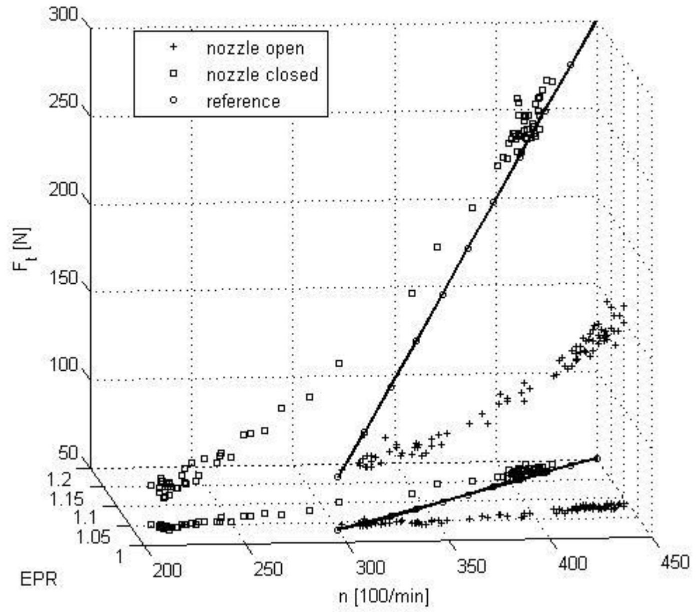

Figure 5. Established reference points plottedrotor speed, EPR, thrust force

5. CONCLUSION

The introduction intended to give a presentation about the latest trends of the aero-engine control. It was also visible, that the aero-engine is a complex system with non-linear characteristics if the whole operating envelope of the engine is considered, thus designing a controller for that is efficiently usable throughout the operating range is a demanding task. The aim of this work is to present a calculation process based on measurement used for determining the reference values of output variables that are applied in a gain scheduling LQI control, that covers the operating range of the investigated variable exhaust nozzle single spool turbojet engine, TKT-1. The calculation process mainly uses linear relationships that are established based on the measurement data, and gas dynamic equations to determine the reference EPR values. The application and results of the calculation process were then explained in chapter 4 of the work. The investigated jet engine has only one operating variable, the throttle lever angle. The main achievement of this work is, that the introduced calculation process can feed a multivariable system with output reference values, namely the rotor speed and engine pressure ratio. Furthermore, the algorithm creates the reference values in such a way, that the thrust force created by the engine changes linearly as the function of the throttle lever angle.

6. LITERATURE LIST

Journals:

[1] Foroozan, Z., Veress, Á.: Derivation and Application of a New Equation for Design and Analysis of Triple Spool Mixed Turbofan Jet Engines with Verification. *Acta Polytechnica Hungarica.* 2016. Vol. 13. No. 6. P. 147-162.

[2] Veress, Á. - Bicsák, Gy. - Rohács., D.: Pressure Loss and Flow Uniformity Analysis of Baseline and Redesigned Engine Inlet Duct for a Small Turboprop Aircraft. *Czech Aerospace Proceedings.* 1/2016. P. 3-9.

[3] Pakhmehr, M. - Fitzgerald, N. - Feron, E. M. - Shamma, J. S. - Behbahani, A.: Gain Scheduling Contrl of Gas Turbine Engines: Stability and Verification. *Journal of Engineering for Gas Turbines and Power*. 2014. Vol. 136. P. 031201-1 - 031201-15. DOI: 10.1115/1.4025637

[4] Shi, Y. - Zhao, J. - Liu. Y.: Switching Control for Aero-Engines Based on Switched Equilibrium Manifold Expansion Model. *IEEE Transactions on Industrial Electronics*. 2016. DOI: 10.1109/TIE.2016.2633470.

[5] Du, X. - Guo, Y. - Sun, H.: An Adaptive Model Predictive Controller for Turbofan Engines. *American Journal of Engineering Research (AJER)*. 2015. Vol. 4. No. 12. e-ISSN: 2320-0847. P. 170-176

[6] Wiese, A. P. - Blom, M. J. - Manzie, C. - Brear, M. J. - Kitchener, A.: Model reduction and MIMO Model Predictive Control of Gas Turbine Systems. *Elsevier Control Engineering Practice*. 2015. Vol. 45. ISSN: 0967-0661. P. 194-206.

[7] Beneda, K. - Andoga, R. - Fozo, L.: Linear Mathematical Model for State Space Representation of Small Scale Turbojet Engine with Variable Exhaust Nozzle. *Periodica Polytechnica, Transporatation Engineering.* 2017. Vol. 10605. DOI: 10.3311/PPtr.10605.

[8] Kamenicky, J. - Sykoroa, Z.: Increased Reliability of Aircraft Engines AI-25TL In L-39. *Acta Avionica*. 2001. Vol 3. No. 3. ISSN 1339-9853 (online). P. 29-38.

[9] Hocko, M. - Urda, V.: Analysis of Unstable Phenomena In Different Parts of the Aviation Jet Engine. *Acta Avionica*. 2012. Vol 14. No 24. ISSN 1339-9853 (online). P. 196-202.

[10] Beneda, K.: Modular Electronic Turbojet Control System Based On TPR. *Acta Avionica*. 2015. Vol 17. No 1. ISSN 1339-9853 (online).

Books:

[11] Nascimento, M. A. R., et al. Micro Gas Turbine Engine: A Review. In: *Progress in Gas Turbine Performance,* edited by Dr. Ernesto Benini, 2013, ISBN: 978-953-51-1166-5, InTech, DOI: 10.5772/54444. Available from: http://www.intechopen.com/books/progress-in-gas-turbine-performance/micro-gas-turbine-engine-a-review

[12] Cohen, H. - Rogers, G. F. C. - Saravanamuttoo, H. I. H.: *Gas Turbine Theory. Fourth edition* Harlow, England: Longman Group Limited. 1996. ISBN 0-582-23632-0. 22 p.

[13] Smil, V.: *Prime Movers of Globalizations: The History and Impact of Diesel Engines and Gas Turbines*. MIT press, 2010. ISBN 0262297043, 9780262297042 132 p.

[14] Kulikov, G. G. - Thompson, H. A.: *Dynamic Modeling of Gas Turbines: Identification, Simulation, Condition Monitoring and Optimal Control.* Springer Verlag, London, 2004. ISBN 1-85233-784-2.

[15] Yahya, S. M.: *Fundamentals of Compressible Flow with Aircraft and Rocket Propulsion. Third Edition.* New Delhi, India. New Age International Ltd. 2003. ISBN 81-224-1468-0. 55-62 p.

[16] Walsh, P. P. - Fletcher, P.: Gas Trubine Performnce. Second Edition. Oxford, England. Blackwell Science Ltd. 2004. ISBN 0-632-06434-X. 116 p.

Conference Proceedings:

[17] Fozo, L. - Andoga, R. - Kovacs, R.: Thermo-dynamic Cycle Computation of a Micro Turbojet engine. *IEEE 17th International Symposium on Computational Intelligence and Informatics (CINTI).* Budapest, Hungary. 2016. P. 000075-000080. DOI: 10.1109/CINTI.2016.7846382

[18] Wang, Y. - Wang, S. - Wang, X. - Shi, J.: Gain Scheduling Controller of the Aero-engine Based on LPV model. *IEEE Chinese Guidance, Navigation and Control Conference (CGNCC).* Nanjing, China. 2016. P. 2265-2270.

[19] Pakhmehr, M. - Fitzgerald, N. - Feron, E. M. - Shamma, J. S. - Behbahani, A.: Gain Scheduling Contrl of Gas Turbine Engines: Stability by Computing a Single Quadratic Lyapunov Function. *Proceedings of the ASME Turbo Expo 2013 (GT2013).* San Antonio, Texas, USA. 2013.

[20] Zhu, K. - Zhao, J. - Dimirovski, G. M.: State Feedback Control for Switched LPV Systems with Actuator Saturation and its Application to Aero-engines. *34th Chinese Control Conference* Hangzhou, China. 2015. P 2224-2228.

[21] Zhu, F. - Sun, X. - Wu. Y.: H_∞ Control for Tracking Targets of Aero-engines. *35th Chinese Control Conference*. Chengdu, China. 2016. P. 5879-5884.

[22] Kim, J. S. - Powell, K. M. - Edgar, T. F.: Nonlinear Model Predictive Control for a Heavy Duty Gas Turbine Power Plant. *American Control Conference (ACC)*. 2013. Washington, DC, USA. P. 2952-2957.

[23] Surendran, S. - Chandrawanshi, R. - Kulkarni, S. - Bhartiya, S. - Nataraj, P. S. V. Sampath, S.: Model Predictive Control of a Laboratory Gas Turbine. *Indian Control Conference*. 2016. Hyderabad, India. P. 79-84.

[24] Davies, C. - Holt, J. E. - Griffin, I. A. Benefits of inverse model control of Rolls-Royce civil gas turbines. *International Control Conference*., 2006. Glasgow, Scotland. ISBN: 0974649549. P. 70-76.

[25] Beneda, K. - Fozo, Ladislav.: Identification of Small Scale Turbojet Engine with Variable Exhaust Nozzle. *IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI).* 2017. Kosice, Slovakia. P. 000073-000078.

[26] Garg, S.: Turbofan Engine Control System Design Using the LQG/LTR Methodology. *IEEE American Control Conference*. 1989. Pittsburgh, Pennsylvania, USA. P. 134-141.

[27] Knoll, Z. - Tao, G.: Multivariable Adaptive LQ Control of Jet Engines. *American Control Conference*. 2015. Chicago, Illinois, USA. P. 1193-1198.

[28] Beneda, K. - Linear Quadratic Integral (LQI) Control for a Small Scale Turbojet Engine with Variable Exhaust Nozzle. *ICMT 6th International Conference on Military Technologies*. 2017. Brno, Checz Republic. In press.