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INTEGRATION ARCHITECTURES OF NAVIGATION SYSTEM

Róbert BRÉDA*, Dávid MEGYESI
Department of Avionics, Faculty of Aeronautics, Technical University in Košice, Slovakia
**Corresponding author*. E-mail: robert.breda@tuke.sk

Summary. The content of the contribution is the integration of INS and GNSS systems. The freely linked INS / GNSS integration architecture uses GNSS position and velocity data as measuring inputs to the integration algorithm, regardless of the type of INS correction or GNSS output implementation. The paper emphasizes the appropriate Kalman filtration and its basic mathematical apparatus. At the end of the paper is a brief overview of the basic equations of inertial navigation.

Keywords: INS, GNSS, Kalman Filter, navigation

1. INTRODUCTION

Modern navigation systems are the result of several years of development of basic navigation principles and techniques. Given the inherent technical characteristics of each application, also in case of the navigation it is necessary to seek continuous improvement processes. Among the many factors affecting the operation, performance, and application characteristics, it is important to eliminate the ones that lead to the reduction of accuracy and safety in the performance of tasks. A process for the improvement of the navigation complexes requires, despite efforts to achieve the best features of a particular unit, sometimes complementary connection to another system in principle. Basic issue of the INS/GNSS integration.

1.1. Basic sensor analysis of INS

The inertial navigation system represents an integration article whose resulting information is based on acceleration and angular velocity measurements. Time changes of parameters (e.g., distances or speeds) are also affected by the sensor's increment of time. The operational characteristics of inertial sensors (accelerometers and gyroscopes) are affected by various errors that are categorized as deterministic and stochastic. Deterministic errors occur systematically, so we can determine their value and minimize these errors appropriately. Based on the correct error determination, we can determine the error model that we will count on when determining the position from the measured inertial sensor signals. Stochastic errors are random errors that occur as a result of a random change of the transmission function (bias / scale factor) of sensor over time, referred to as the drift of these parameters.

$$\delta a^b = N_a a^b + S_a a^b + b + \mu_a + w_a \quad (1)$$
$$\delta \omega_{ib}^b = N_\omega \omega_{ib}^b + S_\omega \omega_{ib}^b + d + \mu_\omega + w_\omega \quad (2)$$

Equations (1) and (2) describe errors of accelerometer and gyroscope. In case (1) N_a describes skew – symmetric matrix of non – ortogonality, S_a is diagonal matrix of scale errors, b (or in (2) identified by d is sensor bias μ_a represents other type of error and w_a is sensor noise. In equation (2) are used the same parts, but for gyroscope description.

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1.2. GNSS errors

In case of the GNSS systems is, as well as in previous case, a lot of error sources, which adversely affect results of the navigation process. Error model of both systems is too coarse for content of this contribution. Using information form [1] and [2], basic errors of GNSS systems are as follows.

Table 1 Basic GNSS errors

Type of error	Typical value [m]
Multipath	2,5
Ionospheric error	4,6
Tropospheric error	1,5
Ephemeris error	4,0
Satellite clock error	1,0
Receiver clock error	1,5

GNSS and INS integration is based on nearly ideal complementary characteristics of both systems. In term characteristics is contained dynamic of the system, their navigation constraints and errors. Very important aspect of GNSS/INS error analysis is also dynamic of the system and bounded or unbounded growth of error in time. Because of the practical issue, some of the applications require mathematical definition of the distance between IMU position and GNSS antenna location. Evaluation of the navigation information can be in area of one object apart of each other, and using [2] definition of this distance is as follows.

$$\Delta_{ant,RPY} = \begin{bmatrix} \Delta_{ant,R} \\ \Delta_{ant,P} \\ \Delta_{ant,Y} \end{bmatrix} \quad (3)$$

Vector determining the variation of the antenna position by Euler angles is further transformed into the NED system (north – east – down). The equations, which determine “the point of evaluation” of the navigation problems, are as follows [2].

$$x_{ant,NED} = x_{IMU,NED} + \Delta_{ant,NED} \quad (4)$$
$$x_{IMU,NED} = x_{ant,NED} - \Delta_{ant,NED} \quad (5)$$

In equation (4) left part represents attitude of the GNSS antenna in NED frame, and in equation (5) left part represents attitude of the IMU in NED frame. Following picture very simply describes the problem of the IMU and antenna location also with their displacement.

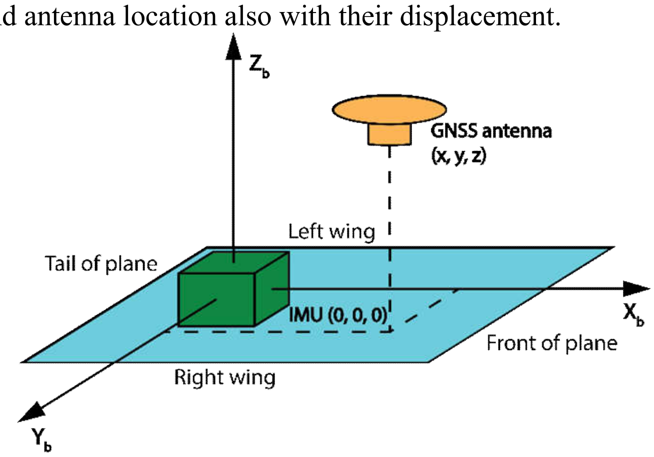

Figure 1 Distance between IMU and GNSS antenna

2. INTEGRATION ARCHITECTURES FOR NAVIGATION

The role of the resulting architecture is to create such mathematical connections that result in a complementarity of the systems in terms of suppressing errors, ensuring appropriate dynamics and safety. Variation formats may vary widely, depending on the number and nature of integrated systems. According to [1], in field of GNSS/INS integration are used for types of integration architecture:

- uncoupled navigation systems
- loosely coupled systems
- tightly coupled systems
- ultra – tightly coupled systems

The binding of systems is characterized by the rate of utilization of information from various systems and their mutual correction. Achieving of the favorable outputs, which are the result of the optimally proposed architecture, depends on the mathematical apparatus. The complexity of this system is, moreover, also based on the used Kalman filtration. The content of this contribution is the description of the loosely coupled integration architecture and using [3], the block diagram of this architecture is as follows.

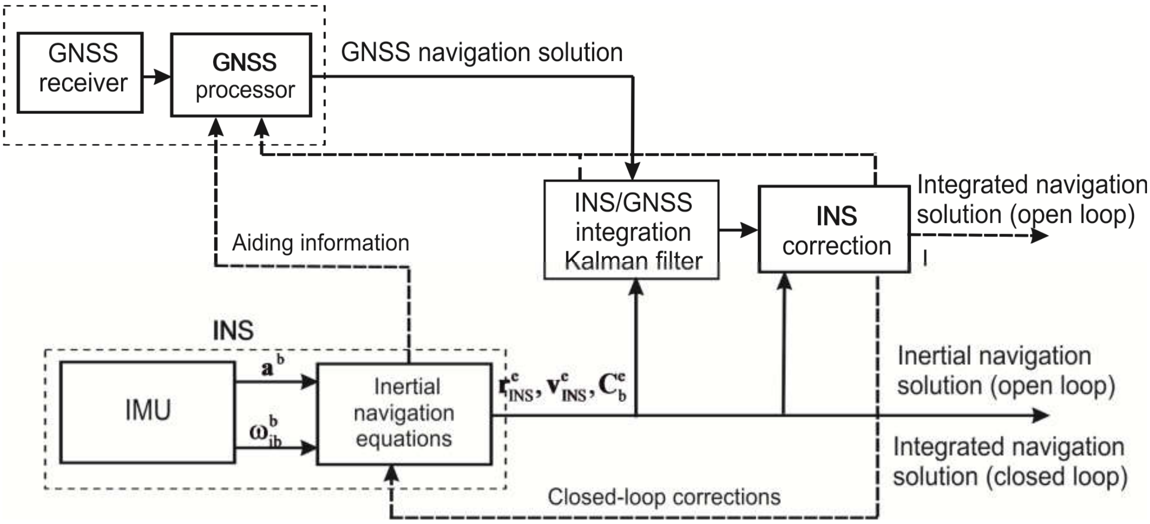

Figure 2 Loosely coupled INS/GNSS integration architecture

This integration architecture uses essentially two Kalman filters, the characteristic of the architecture is transferred mainly to the integration Kalman filter. Most authors consider to applying of this estimation algorithm uses the following vector measurements, like in [2].

$$z_{GNSS} = h_{GNSS}(x_{veh}) + v_{GNSS} \tag{6}$$
$$z_{INS} = h_{INS}(x_{veh}) + v_{INS} \tag{7}$$

These equations are used in following parts of the contribution, where z represents the measurement vector, h is the function of measurements and x is the state vector of object attitude. Measurement noise is, in accordance with the theory of Kalman filtering, the additive part of the measurements. In case of system integration is important to know the basic characteristics of both systems. The INS is characterized by high information dynamics and small error in short intervals of time. Its drawback is unlimited growth of errors in time and also the need for correction of the impact of gravity. By contrast, the field of the GNSS systems is characterized by time – limited error with lower information dynamics of output information. If we choose GPS as the GNSS system, we will assume frequencies 1575,42 MHz and 1227,6 MHz . Operating frequency of IMU is one of the variable parameters, in the case referred to [4], [10] and [11] is bandwidth of used sensors 0,33 kHz.

3. KALMAN FILTER FOR LOOSELY COUPLED INTEGRATION ARCHITECTURE

Problems of Kalman Filter in terms of mathematical principles, statistical characteristics and the equations are part of numerous publications and scientific articles [2], [6]. This estimation algorithm is one of the problematic elements in this architecture because of the cascade application. The following section addresses directly with the equations of Kalman filter, whose detailed description is given in [6], [9], [12]. Characteristically for this integration architecture is used nine state Kalman filter, which is based on the equations of INS error dynamics. The basic is perturbation method for linearization of the nonlinear differential equations. For derivation is applied perturbation of position, velocity, direction – cosine matrix and gravitational parameters. If we use a description of the system state in a continuous time:

$$\dot{x} = \boldsymbol{F}x + \boldsymbol{G}u \tag{8}$$

In equation (8) F represents matrix of system dynamics and G is matrix of input coupling. Following mathematical description represented by matrices and vectors of the Kalman filter is suitable for application in field of the loosely coupled architecture [7].

$$F = \begin{pmatrix} F_{rr} & F_{rv} & 0 \\ F_{vr} & F_{vv} & (f^n \times) \\ F_{er} & F_{ev} & -(\omega_{in}^n \times) \end{pmatrix} \tag{9}$$

In matrix (9) are used elements, which are widely described in [1], [7].

$$x = \begin{pmatrix} \delta r^n \\ \delta v^n \\ \epsilon^n \end{pmatrix} \tag{10}$$

The state vector (10) contains nine elements, δr^n represents error in attitude, δv^n express velocity error and ϵ^n express error in angle attitude.

$$G = \begin{pmatrix} 0 & 0 \\ C_b^n & 0 \\ 0 & -C_b^n \end{pmatrix} \tag{11}$$

The content of G matrix is transformation matrix from body frame to navigation frame.

$$u = \begin{pmatrix} \delta f^b \\ \delta \omega_{ib}^b \end{pmatrix} \tag{12}$$

The last part of equation (12) is vector u which consists of two error sources. First one is accelerometer error computed in body frame and the second one is gyroscope error also in body frame. For prediction and correction equations of Kalman filtering is also necessary evaluation of process noise covariance matrix Q and measurement noise covariance matrix R. Simplified mathematical form can be as follows.

$$Q = diag\begin{pmatrix} \sigma_{ax}^2 & \sigma_{ay}^2 & \sigma_{az}^2 & \sigma_{\omega x}^2 & \sigma_{\omega y}^2 & \sigma_{\omega z}^2 \end{pmatrix} \tag{13}$$

$$R = diag\begin{pmatrix} \sigma_{\varphi}^2 & \sigma_{\lambda}^2 & \sigma_{h}^2 & \sigma_{vn}^2 & \sigma_{ve}^2 & \sigma_{vd}^2 \end{pmatrix} \tag{14}$$

Program implementation of these equations is based on suitable form of discretization and in this case is used Van Loan method. Evaluation of state transition matrix from matrix F is based on following expression.

$$\phi_k = \exp(F\Delta t) \approx I + F\Delta t \tag{16}$$

In equation (16) I represents the identity matrix and Δt discrete time interval. This discretization method is used also for evaluation of discrete covariance matrix Q. In this case is used following equation.

$$Q_k \approx \phi_k G Q G^T \phi_k^T \Delta t \tag{17}$$

All parts of equation (17) are explained in previous part of this section.
For example, using equations (16) and (17), difference between F and ϕ matrix is as follows.

Table 2 Parameters necessary for F matrix evaluation

ω_{in}^{nx} [°/s]	0,001
ω_{in}^{ny} [°/s]	0
ω_{in}^{nz} [°/s]	0
v^e [m/s]	2
v^n [m/s]	30
v^d [m/s]	0
$\varphi_{latitude}$ [°]	50
f_x [m/s^{-2}]	0,1
f_y [m/s^{-2}]	0,015
f_z [m/s^{-2}]	0
ψ^b [°]	6
ϕ^b [°]	10
θ^b [°]	15

The process of discretization in case of discrete process noise covariance matrix also requires the value of the discrete time interval Δt, which is the same as in previous case of STM evaluation.

The discrete equation of measurement for Kalman filter are based on (8) and have following form.

$$z_k = \begin{pmatrix} r_{INS}^n - r_{GNSS}^n \\ v_{INS}^n - v_{GNSS}^n \end{pmatrix} \tag{18}$$

Following the basic idea of integration architecture expressed by (6) and (7), discrete measurement of Kalman filter represents in first part error between attitude of INS and GNSS system. Second part also represents error, but in velocity evaluated by both systems.

4. EQUATION FOR INS ACTUALIZATION

As well as other integration architectures, also loosely coupled architecture represents mathematical solution in order of program implementation of mathematical models and measurements of individual systems. As well as in case of Kalman filter, for field of navigation systems integration are necessary discrete forms of evaluation and actualization of all parameters. In range of this contribution with knowledge from [7], [8] are chosen some parameters of integration architecture. In terms of transformation is necessary discrete actualization of direction cosine matrix, which is such as all of the following widely described in [7] and [8].

The DCM can be described as mathematical tool for transformation from one frame to another. Including dynamics of moving object, it is necessary to express rotation of one frame with respect to another. This rotation is evaluated by angular rate ω_{ab}^{b}, where a and b are frames. Using [8], equation of the rotation vector rate $\dot{\phi}$ is as follows.

$$\dot{\phi} = \omega_{ab}^{b} + \frac{1}{2}\phi \times \omega_{ab}^{b} + \frac{1}{12}\phi \times \left(\phi \times \omega_{ab}^{b}\right) \tag{19}$$

Using this value of rotation vector, evaluation of the transformation matrix is as follows.

$$C_b^a = I + \frac{sin\,\|\phi\|}{\|\phi\|}(\phi\,\times) + \frac{1 - cos\,\|\phi\|}{\|\phi\|^2}(\phi\,\times)^2 \tag{20}$$

In equation (20) is used skew - symmetric matrix symbol, which matrix form is:

$$(\phi\,\times) = \begin{bmatrix} 0 & -\phi_z & \phi_y \\ \phi_z & 0 & -\phi_x \\ -\phi_y & \phi_x & 0 \end{bmatrix} \tag{21}$$

For discrete expression of the velocity actualization is derived equation (22), which parts are specified in [7], [8].

$$v_k^n = v_{k-1}^n + \Delta v_{f,k}^n + \Delta v_{\frac{g}{cor},k}^n \tag{22}$$

Equation (22) is the discrete form of continuous time form of the velocity evaluation derived . In comparison with continuous form, which parts are transformation, centripetal and Coriolis forces, this one has previous value of the discrete velocity, increment of the specific force and the last part is increment of the gravity and Coriolis force. Following equation express the evaluation of the specific force increment.

$$\Delta v_{f,k}^n = \frac{1}{2}\left[C_{n(k-1)}^{n(k)} + I\right] C_{b(k-1)}^{n(k-1)} \Delta v_{f,k}^{b(k-1)} \tag{23}$$

Equation (23) is based on iterations of the transformation matrix C_b^n and the velocity increment $\Delta v_{f,k}^b$.

$$\Delta v_{\frac{g}{cor},k}^n = \left[g^n - (2\omega_{ie}^n + \omega_{en}^n) \times v^n\right]_{k-\frac{1}{2}} \Delta t_k \tag{24}$$

In equation (24), part g^n is normal gravity, often marked as γ^n. Evaluation of this variable is through following equation (25). The equation of the gravity and Coriolis force consists of the angular rate between earth and inertial navigation frame ω_{ie}^{n} and angular rate between navigation and earth frame ω_{en}^{n}.

$$\gamma^n = (0\ 0\ \gamma)^T \tag{25}$$

The last part of the vector (25) is evaluated by following equation.

$$\gamma = a_1(1 + a_2 sin^2\varphi + a_3 sin^4\varphi) + (a_4 + a_5 sin^2\varphi)h + a_6 h^2 \tag{26}$$

Used variables are height above ellipsoid h and latitude φ. The values $a_1 - a_6$ are constants from model WGS – 84.

The discrete actualization of position is expressed using quaternions.

$$q_{n(k)}^{e(k-1)} = q_{n(k-1)}^{e(k-1)} * q_{n(k)}^{n(k-1)} \tag{27}$$

$$q_{n(k)}^{e(k)} = q_{e(k-1)}^{e(k)} * q_{n(k)}^{e(k-1)} \tag{28}$$

Parts of equations (27) and (28) are expressed by these quaternions.

$$q_{n(k)}^{n(k-1)} = \begin{bmatrix} cos\|0.5\xi_k\| \\ \dfrac{sin\|0.5\xi_k\|}{\|0.5\xi_k\|}0.5\xi_k \end{bmatrix} \tag{29}$$

$$q_{e(k-1)}^{e(k)} = \begin{bmatrix} cos\|0.5\xi_k\| \\ -\dfrac{sin\|0.5\xi_k\|}{\|0.5\xi_k\|}0.5\xi_k \end{bmatrix} \tag{30}$$

Quaternions expressed by (29) and (30) consist value ξ_k. Using [1], this value is described as rotation vector based on angular rate ω_{ie}^{e}.

Actualization of attitude using quaternions has following form.

$$q_{b(k)}^{n(k-1)} = q_{b(k-1)}^{n(k-1)} * q_{b(k)}^{b(k-1)} \tag{31}$$

$$q_{b(k)}^{n(k)} = q_{n(k-1)}^{n(k)} * q_{b(k)}^{n(k-1)} \tag{32}$$

Parts q_b^b and q_n^n are expressed by following equations:

$$q_{b(k)}^{b(k-1)} = \begin{bmatrix} cos\|0.5\phi_k\| \\ \dfrac{sin\|0.5\phi_k\|}{\|0.5\phi_k\|}0.5\phi_k \end{bmatrix} \tag{33}$$

$$q_{n(k-1)}^{n(k)} = \begin{bmatrix} cos\|0.5\zeta_k\| \\ -\dfrac{sin\|0.5\zeta_k\|}{\|0.5\zeta_k\|}0.5\zeta_k \end{bmatrix} \tag{34}$$

In quaternion (34) is used ϕ_k, which is known from (22), and value ζ_k expressed as:

$$\zeta_k = (\omega_{ie}^n + \omega_{en}^n)_{k-\frac{1}{2}} \Delta t_k \qquad (35)$$

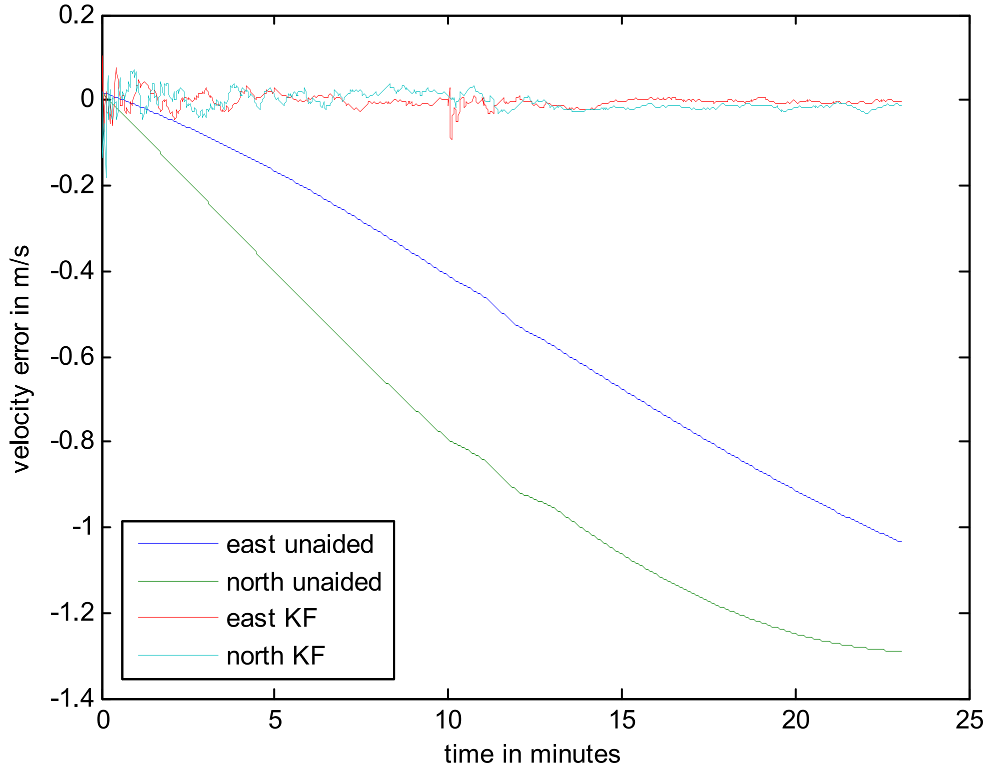

Figure 3 Introducing a speed error estimate as a result of Linearized Kalman Filter

5. CONCLUSION

The structure is based on loosely coupled architecture of the navigation systems integration. The Kalman filtering is also important part, and in given range is derived Linearized Kalman Filter for chosen architecture. For better explanation is shown discretization of this estimation algorithm, especially discretization of the chosen matrices F and Q. The last part of the contribution is focused on basics of the real time algorithms of INS. The real time application of INS equations is closely related with program implementation of this problem on board of flying object. The goal of this contribution is to summarize the main points of the function and program implementation issue. Given overview of the mathematic apparatus also with used references create an appropriate basis for unmanned applications. In terms of Kalman filter application is also necessary the analysis of the sensor unites, which create the values of the covariance matrices of this estimation algorithm.

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