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THE FORMULAS

Peter SZABÓ*
Technical University of Košice, Faculty of Aeronautics
Department of Aviation Technical Studies, Rampová 7, 040 01 Košice, Slovak Republic
Miroslava FERENCOVÁ
Technical University of Košice, Faculty of Aeronautics
Department of Air Transport Management, Rampová 7, 040 01 Košice, Slovak Republic
**Corresponding author.* E-mail: peter.szabo@tuke.sk

Abstract. The most effective way to express the mathematical and physical research results is by using the formulas. Our article contains the formulas that were the biggest contributors to the development of technical, physical, and mathematical sciences. The formulas can be applied in different research areas. In this article, we will show how to calculate an orbit of a miniature satellite based on the Law of Gravity. For calculations and online recording of results, we are using the MATLAB system. This article contains also a description of some formulas discovered by the authors.

Keywords: mathematical formulas; physical formulas

1. INTRODUCTION

Science is the cornerstone of civilization. We capture the knowledge of the world around us; consequently, we pass this knowledge on to the next generation. Scientific knowledge present solved problems that we can express as procedures, methods, algorithms. There are diverse areas of knowledge; they vary from exploring the oceans to flying and discovering space, but there are also medical procedures that are used to cure diseases. Algorithms and procedures can be expressed most succinctly using formulas that contain mathematical operations and relationships between numbers that can be concrete or abstract.

2. NUMBERS

Numbers play a very important role in recording procedures. By using numbers, we can express counts, quantities, sizes and quantify the examined objects. Numbers can be both concrete and abstract. The concrete numbers are constants, e.g. $\pi = 3.1415....$ is *Ludolph's number* - it is the ratio of the circumference of an arbitrary circle to its diameter. If we denote the circumference of the circle as O and the diameter as r, the constant π can be expressed as

$$\pi = \frac{O}{2r} \tag{1}$$

Numbers O, r are no longer concrete numbers. They are *variables*, *symbols, abstract numbers* for which we can place the values of an arbitrary circle. Relations of the type (1) are called *formulas*. A formula is an expression that contains mathematical operations and expresses relations between variables (O, r) and constants (π), that describe a given problem. Formulas express scientific knowledge in the most concise way. Formulas can be mathematical, physical, chemical, and biological. It is to be noted that in each field of science there are special constants. The system of constants used in astronomy can be found in the publication [3].

3. FORMULAS

In this chapter, we will list some of the formulas that have had a major impact on the development of scientific knowledge and we may encounter them in the study of mathematics, physics but also in other scientific disciplines. The formulas can be found in publication [5]. This publication contains 17 of the most important formulas with a description of the historical background of their origin. In this article, we list 8 of them.

Table 1 Formulas - results of research knowledge

1	$a^2 + b^2 = c^2$	Pythagoras theorem	Pythagoras, Bc 560
2	$log(xy) = log(x) + log(y)$	Logarithm	J. Napier, 1610
3	$\frac{df}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$	Derivative, differential calculus	I. Newton, 1668
4	$F = G\frac{m_1 m_2}{r^2}$	Newton's Law of Gravity	I. Newton, 1687
5	$i^2 = -1$	Complex numbers	L. Euler, 1750
6	$\Phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$	Normal distribution	C.F. Gauss, 1810
7	$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i \omega x} dx$	Fourier Transform	J. Fourier, 1822
8	$E = mc^2$	Relativity	A. Einstein, 1905

3.1. Mathematical formulas

Mathematical formulas contain mathematical constants, variables, and operations. All formulas (except 4 and 8) are mathematical formulas; formulas 4 and 8 are physical. Mathematical formulas contain only mathematical constants, variables. The number π is a mathematical constant because it was formed using mathematical objects (the circle, the circumference of the circle, and the diameter). Mathematical formulas are universal, they have universal validity.

3.2. Physical formulas

Formulas 4 and 8 are physical because the variables and some constants in relations are physical and we express those using physics. Such a constant is e.g. the gravitational constant G in relation 4, it is one of the fundamental constants of physics [3]. Each formula in the table has been registered in some way. Formula 3 which can be considered as the beginning and basic relation of the differential calculus can be found in [4].

4. APPLICATION OF FORMULAS

The formulas listed in Table 1 have been and are being applied for calculations in various fields of science. In our example, there is an application of the Law of Gravity (formula 4) to calculate the orbit of a miniature satellite. The Law of Gravity was also expressed using Kepler's Laws, which originated in the early 17th century (around 1610). The final formulaic form of the Law of Gravity was developed after 200 years. Kepler's third law describes the relationship between the orbital period of the planets around the Sun and the longitude of the major semi-axis of their elliptical orbits. The generalized form of these laws we have applied to the calculation of the orbit of a miniature satellite around the Earth in [8]. Specific calculations and applications are contained in the program [7].

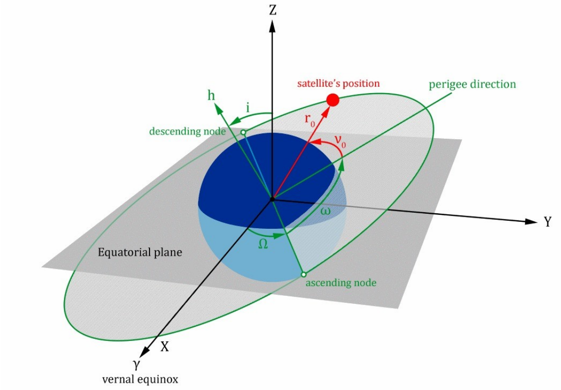

Figure 1 The orbit of a miniature satellite around the Earth - main parameters

5. THE AUTHOR'S FORMULAS

The book [3] inspired us to write some of our scientific results using formulas. Of course, the meaning of these results is not comparable to the results described in the book [3]. Our results are less significant. For a simpler understanding of our formulas, individual chapters contain a brief description of the problems addressed.

5.1. Strong linear independence in bottleneck algebra

The paper [1] deals with the solvability of the system of equations $Ax = b$ in the Bottleneck algebra. By Bottleneck algebra, we mean an algebra in which the basic arithmetic operations of linear algebra (+, ×) are replaced by the operations (⊕, ⊗) = (maximum, minimum). Here we have addressed the question: when does a system of equations

$$A \otimes x = b \tag{2}$$

has a unique solution or in other words, when the matrix A is strongly regular in the Bottleneck algebra. We find that in this algebra, the class of strongly regular matrices consist of trapezoidal matrices. The real square matrix $A = (a_{ij})$ of type $n \times n$ is trapezoidal when

$$a_{ii} > \sum_{k=1}^{i}{}^{\oplus} \sum_{l=k+1}^{n}{}^{\otimes} a_{kl} \tag{3}$$

for each $i = 1, \cdots, n$. This is the formula that describes the problem. The question of testing condition (3), where we can convert the input matrix to trapezoidal form, is a question of algorithms of complexity $O(n^2 logn)$. We have also presented such an algorithm in our work. This work is the result of the cooperation of three authors.

5.2. Monge Matrices

Monge matrices were already mentioned in 1781. The square matrix $A = (a_{ij})$ of type $n \times n$ has the Monge's property when

$$a_{ii} + a_{jk} \leq a_{ij} + a_{ki} \tag{4}$$

for each $i = 1,...,n$ and for every $j, k > i$. The problem of when it is possible to transform a matrix A into a matrix with the Monge's property has not been solved yet. The first solution to this problem is given in [2]. The solution to the problem, when we can transform ($\leftrightarrow$) an input matrix A into a matrix which has the Monge property is the existence of permutations σ, π such that

$$a_{\sigma(i)\pi(i)} + a_{\sigma(j)\pi(k)} \leq a_{\sigma(i)\pi(j)} + a_{\sigma(k)\pi(i)} \tag{5}$$

for each $i = 1,...,n$ and for every $j, k > i$. We can write this attribute as

$$A \Leftrightarrow \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, A_{11} = (a_{\sigma(1)\pi(1)}) \tag{6}$$

This relationship allows us to apply an iterative algorithm to test the relation $\leftrightarrow$ (The relation means that we can transform the matrix A into a Monge matrix if and only if we know the matrix A_{22} and for the element $\alpha_{\sigma(1)\pi(1)}$ the following relation applies (5)). For testing this property of matrices we can use an algorithm of complexity $O(n^4)$. Therefore, it was not easy to find a testing algorithm or a solution to the problem. Note that even the more complicated polynomial matrix algorithms have a complexity no greater than $O(n^3)$. This work is the result of the cooperation of two authors.

5.3. Goldbach's Conjecture in Max-Algebra

In max-algebra, the basic arithmetic operations of linear algebra (+, ×) are replaced by the operations ($\oplus$, $\otimes$) = (maximum, +). The well-known Goldbach conjecture states that every even integer greater than 2 can be expressed as the sum of two primes. So far, no one has proved this nearly 300-year-old conjecture. In max algebra, we can successfully solve some polynomially indecomposable linear algebra problems (e.g. the problem of computing an eigenvalue and an eigenvector). In [6] Goldbach's conjecture in max-algebra is pronounced. To the conjecture, we can associate the formula

$$\frac{t_p + t_q}{3} = \lambda(A) \tag{7}$$

Such formulas have a symbolic meaning. The exact mathematical meaning of each object needs to be defined; in our case, the matrix A, vector t and the eigenvalue $\lambda(A)$. Detailed definitions, theorems, and proofs are found in the work [6]. To illustrate, we state that matrix A is of the form

$$A = \begin{pmatrix} t_0 & 0 & 0 & \dots & 0 \\ t_1 & t_0 & 0 & & 0 \\ t_2 & t_1 & t_0 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ t_{n-1} & & \dots & t_1 & t_0 \end{pmatrix} \tag{8}$$

where vector $t = (t_0, t_1, ..., t_{n-1})^T$ is defined such that $t_i = 0$ a $t_j = 1$ just when j is prime, for each $i, j = 0,1,...,n-1$.

6. CONCLUSIONS

In this paper, we wanted to highlight the importance of mathematical and physical formulas in engineering and mathematical sciences. The most important formulas in these fields can be found in the publication [5]. There are also special constants included in the formulas. Constants used in astronomy are found in publication [3]. These physical constants, like the number π, are probably irrational numbers. That is why the database of physical constants is occasionally updated. The paper also contains some formulas discovered by the authors (3, 6, 7).

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