GRIDS ON THE SURFACE OF REVOLUTION

Tatiana Olejníková

This paper describes a modelling of grids on the sinusoidal surface of revolution. The grids are created by right-handed and left-handed cyclical helical surfaces and by cyclical surfaces with axis on meridians and circles on the sinusoidal surface.

K e y w o r d s: cyclical helical surface, surface of revolution, grid, meridian

1 INTRODUCTION

Let the three-dimensional Euclidean space E^3 is determined by the Cartesian coordinate system $(0, x, y, z)$. The sine curve k determined by parametric equations

$$x_k = a_1 + a_2 \sin v,\ y_k = 0,\ z_k = a_3 v,\ v \in \langle 0, 3\pi \rangle, \qquad (1)$$

for parameters $a_1 = 7,\ a_2 = 3,\ a_3 = 3$, rotates about coordinate axis z and creates the surface of revolution Φ determined by the vector function

$$\mathbf{P}(u, v) = (x_k \cos u,\ x_k \sin u,\ z_k, 1),\ u \in \langle 0, 2\pi \rangle. \qquad (2)$$

Let´s define the helix s on the surface Φ by the vector function

$$\mathbf{r}(v) = (x_s, y_s, z_s, 1) = (x_k \cos mv,\ \mathrm{sgn}\ x_k \sin mv,\ z_k, 1), \qquad (3)$$

for the parameter $v \in \langle 0, 3\pi \rangle$, the parameter sgn determines the helix orientation ($\mathrm{sgn} = +1$ for the right-handed and $\mathrm{sgn} = -1$ for the left-handed helical movement), the parameter m determines an angular velocity of the helical movement of the point on the surface Φ. We can create the grid of n right-handed and n left-handed helices on this surface Φ determined by equations

$$\begin{aligned} x(i) &= x_s \cos i\alpha - \mathrm{sgn}\ y_s \sin i\alpha, \\ y(i) &= x_s \sin i\alpha + \mathrm{sgn}\ y_s \cos i\alpha, \\ z(i) &= z_s \end{aligned} \qquad (4)$$

where $i = 1, \ldots, n$ and the angle $\alpha = 2\pi / n$.

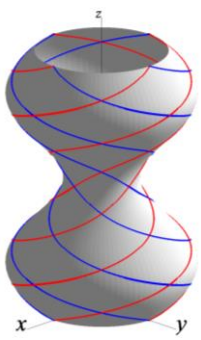

Fig. 1

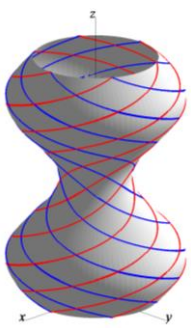

Fig. 2

In Fig.1 is displayed the surface Φ with the grid for $n = 4$ right-handed and $n = 4$ left-handed helix determined by the parameter $m = 1$, in Fig.2 for $n = 6$ are pairs of helices on the surface Φ.

2 MODELING OF CYCLICAL HELICAL SURFACES

Let $(0', n, b, t)$ be the Frenet-Serret moving trihedron of the helix s represented by the regular square matrix

$$\mathbf{M}(v) = \begin{pmatrix} n_x(v) & n_y(v) & n_z(v) & 0 \\ b_x(v) & b_y(v) & b_z(v) & 0 \\ t_x(v) & t_y(v) & t_z(v) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (5)$$

where matrix elements are coordinates of the unit vectors of the principle normal n, binormal b and tangent t of the helix s at the point $0' \in s$ in the coordinate system $(0, x, y, z)$

$$\mathbf{t}(v) = (t_x(v), t_y(v), t_z(v)) = \frac{\mathbf{r}'(v)}{|\mathbf{r}'(v)|}, \qquad (6)$$

$$\mathbf{b}(v) = (b_x(v), b_y(v), b_z(v)) = \frac{\mathbf{r}'(v) \times \mathbf{r}''(v)}{|\mathbf{r}'(v) \times \mathbf{r}''(v)|}, \qquad (7)$$

$$\mathbf{n}(v) = (n_x(v), n_y(v), n_z(v)) = \mathbf{b}(v) \times \mathbf{t}(v). \qquad (8)$$

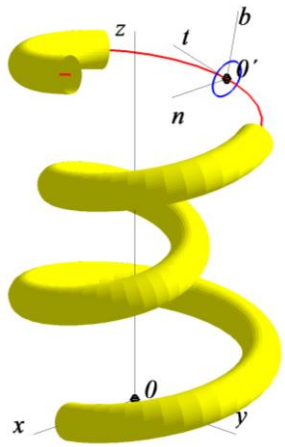

Fig. 3

Let the circle $c = (0', r)$ with its center $0'$ and the radius r moves along the helix s and lies in the normal

plane determined by the principal normal n and the binormal b of the helix s at the point $0'$ and creates the cyclical surface Φ' (Fig.3). Then the vector function of this cyclical surface Φ' is

$$\mathbf{P}'(u,v)=\mathbf{r}(v)+\mathbf{c}(u).\mathbf{M}(v),\ u\in\langle 0,2\pi\rangle,\ v\in\langle 0,3\pi\rangle, \quad (9)$$

where $\mathbf{r}(v)$ is vector function of the helix s expressed in equations (3), $\mathbf{M}(v)$ is the transformation matrix of the coordinate system $(0',n,b,t)$ into the coordinate system $(0,x,y,z)$, (Eq.5) and $\mathbf{c}(u)=(r\cos u,\ r\sin u,\ 0,1)$, $u\in\langle 0,2\pi\rangle$ is the vector function of the circle c.

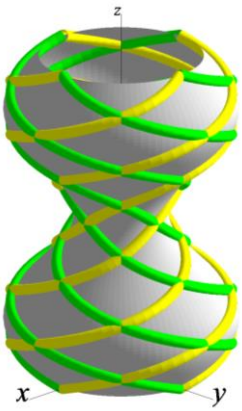

Fig. 4

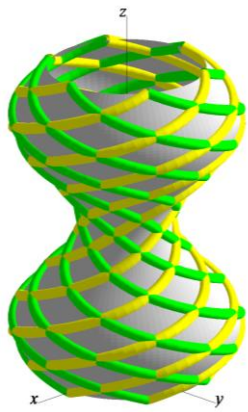

Fig. 5

In Figs.4,5 are displayed the surface Φ and $2n$ surfaces Φ' for parameters $a_1=7, a_2=3, a_3=3$, $m=1, n=4$ or $n=6$.

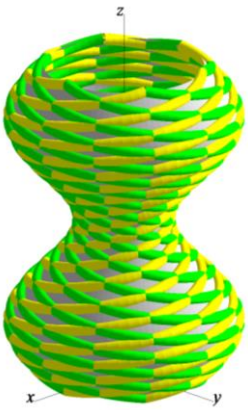

Fig.6

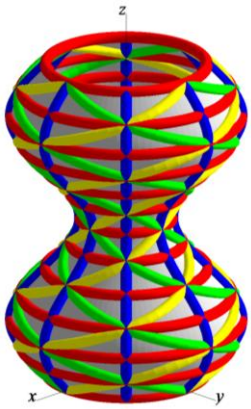

Fig.7

Fig.6 shows the surface Φ and $2n$ surfaces Φ', for parameters $a_1=7, a_2=3, a_3=3, m=2, n=6$.

In Fig.7 are displayed the surface Φ, n right-handed, n left-handed cyclical helical surfaces Φ' and $2n$ cyclical surfaces with axis on meridians of the surface Φ determined by parametric equations

$$x(\mathrm{i})=x_k\cos\mathrm{i}\alpha/2,\ y(\mathrm{i})=x_k\sin\mathrm{i}\alpha/2,\ z(\mathrm{i})=z_k, \quad (10)$$

and $3n$+1 cyclical surfaces on the circles with parametric equations

$$x(\mathrm{i})=r(i)\cos u,\ y(\mathrm{i})=r(i)\sin su,\ z(\mathrm{i})=\mathrm{i}\,a_3\pi/4,\ u\in\langle 0,2\pi\rangle \quad (11)$$

where the radius of circle is $r(\mathrm{i})=a_1+a_2\sin\mathrm{i}\pi/4$.

3 COMPOSED CYCLICAL HELICAL SURFACES

In this chapter we create the cyclical surface Φ'' by the movement of the circle $c'=(0'',r')$ with the center $0''$ and the radius r' along the helix s', which is created by the rotation of the point $(x_0,y_0,z_0,1)$ about the tangent t of the helix s. The vector function of this helix s' is

$$\mathbf{r}'(v)=\mathbf{r}(v)+(x_0,y_0,z_0,1).\mathbf{M}(v). \quad (12)$$

Let $(0'',n',b',t')$ be the Frenet-Serret moving trihedron of the helix s' represented by the regular square matrix

$$\mathbf{M}'(v)=\begin{pmatrix} n'_x(v) & n'_y(v) & n'_z(v) & 0\\ b'_x(v) & b'_y(v) & b'_z(v) & 0\\ t'_x(v) & t'_y(v) & t'_z(v) & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \quad (13)$$

where matrix elements are coordinates of unit vectors of the principle normal n', binormal b' and tangent t' of the helix s' at the point $0''\in s'$ in the coordinate system $(0,x,y,z)$

$$\mathbf{t}'(v)=\frac{\mathbf{r}'(v)'}{\left|\mathbf{r}'(v)'\right|}, \quad (14)$$

$$\mathbf{b}'(v)=\frac{\mathbf{r}'(v)'\times\mathbf{r}'(v)''}{\left|\mathbf{r}'(v)'\times\mathbf{r}'(v)''\right|}, \quad (15)$$

$$\mathbf{n}'(v)=\mathbf{b}'(v)\times\mathbf{t}'(v). \quad (16)$$

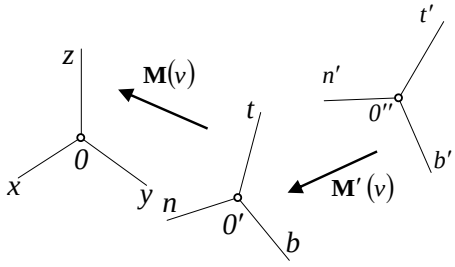

Fig. 8

In Fig.8 are displayed transformations of the coordinate system $(0',n,b,t)$ (the moving trihedron of the

helix s) into coordinate system $(0,x,y,z)$ and the transformation of the coordinate system $(0'',n',b',t')$ (the moving trihedron of the helix s') into the coordinate system $(0',n,b,t)$.

Let the moving circle $c'=(0'',r')$ along the helix s' lies in the normal plane determined by the principal normal n' and binormal b' of the helix at the point $0''\in s'$ create the cyclical surface Φ''. Then the vector function of this cyclical surface Φ'' is

$$\mathbf{P}''(u,v)=\mathbf{r}'(v)+c'(u).\,\mathbf{M}'(v),\, u\in\langle 0,2\pi\rangle,\, v\in\langle 0,3\pi\rangle, \quad (17)$$

where $\mathbf{r}'(v)$ is the vector function of the helix s' expressed in equation (12), $\mathbf{M}'(v)$ is the transformation matrix of the coordinate system $(0'',n',b',t')$ into the coordinate system $(0,x,y,z)$, (Eq.13) and $c'(u)$ is the vector function of the circle determined by its center $0''$ and radius r'.

In Fig.9 is displayed for $n=4$ pairs of right-handed and left handed cyclical surfaces Φ' and cyclical sutfaces Φ'' with the angular velocity $m'=15m$, $\mathrm{sgn}'=+1$. These surfaces are consistently oriented with right-handed cyclical surfaces Φ' and oppositely oriented with left-handed cyclical surfaces Φ'. In Fig.10 is displayed the detail of composed cyclical surfaces of previous figure.

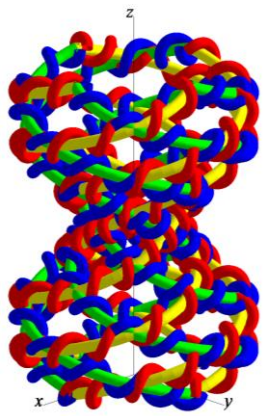

Fig. 9

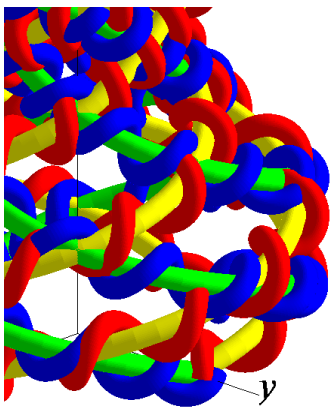

Fig. 10

5 CONCLUSION

Using of the mathematical apparatus we can create spatial ornaments as an example of the using of mathematics in design. In addition to the efficiency in the design is needed as well as aesthetics.

The described method of the modeling of cyclical surface grids on the surface of revolution shows a simply way to modeling of different interesting grids of surfaces by changing of parameters.

Acknowledgements

This work was supported by VEGA 1/0321/12 „Theoretical and experimental analysis of adaptive cable and tensegrity systems under static and dynamic stress considering the effect of wind and seismic“.

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AUTHOR´S ADDRESS

Olejníková Tatiana, RNDr., PhD.
Department of Applied Mathematics
Faculty of Civil Engineering
Technical University of Košice
Vysokoškolská 4, 042 00 Košice
e-mail: tatiana.olejnikovva@tuke.sk