

## VERIFICATION OF SELECTED MODEL COEFFICIENTS FOR A SMALL UNMANNED AIRCRAFT

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**Abstract.** This article presents design and analysis of mathematical model UAV with fixed wings. The mathematical model serves to understand the basic mathematical principle and physical laws, that are applied to the system, and is indispensable in UAV movement simulation and modelling of the control algorithms. The article also describes the methodology of individual modelled parts using the XFLR5, where effective numerical methods are used for mathematical modelling. The last part provides verified and reliable results obtained by simulations of a mathematical model, which will be used to simulate basic and critical situation states.

**Keywords:** mathematical model; UAV; aerodynamic design; static stability

### 1. INTRODUCTION

At present, UAV have a very wide range of uses. They are used for military and civilian purposes, but also for commercial purposes such as science and research. [1] It is in the field of research that they are making significant progress. It also applies to small unmanned aircraft with fixed wings. UAV stabilization is a set of devices and algorithms implemented on board an aircraft to ensure stable and safe flight. The complexity of the system thus requires an adequate mathematical model and its detailed description [2], [3]. All analytical methods and calculation of dynamic characteristics is demanding and therefore requires special solutions. Therefore, it is necessary to build syntheses to solve the assigned tasks and analyzes [4],[5].

The focus of this research is creation and subsequent verification of a test platform for design of control algorithms, which is implemented in simulation environment using 6DoF modeling. The created mathematical-simulation model describes behavior of the UAV, and dynamics of the motion the real aircraft model. It is used in a wide range of simulations, testing, and verification of flight data in both normal and critical situations.

### 2. OBJECT OF THE RESEARCH

Object of the research is a small unmanned aircraft with fixed wings shown in Fig. 1. During its operation, UAV moves in six degrees of freedom(6DoF). Three degrees of freedom are presented as translational angles, which determine the trajectory movement of the aircraft, and the remaining three degrees of freedom represent the position angles, orientation of the aircraft in space such as the roll and pitch.



Figure 1 UAV with fixed wings Carbon Cub 15cc

Table 1 Technical parameters of UAV Carbon Cub 15 cc

<b>Length</b>	1530 mm
<b>Weight</b>	5200 g
<b>Wingspan</b>	2280 mm
<b>Wing loading</b>	98 g/cm <sup>2</sup>
<b>Wing area</b>	0.755 m <sup>2</sup> /s
<b>Engine type</b>	Electro motor AXI5325/18
<b>Battery</b>	Li-Po 6 channels, 22.2 V, 4000 mAh
<b>Propeller</b>	APC 19x10 E

### 3. MATHEMATICAL MODELLING OF UAV

The mathematical model explains the movement and behavior of small unmanned aircraft with fixed wings, with respect to the input values entering system as well as external influences on the UAV. The model can predict the UAV position in three basic flight modes and allows simulation environment UAV behavior in different flight conditions [6]. Simulation is a simple method of verifying and testing algorithms, while preventing damage to real UAV. In general, the aircraft has 6 DoF with non-linear behavior. The aircraft can thus be modelled by nonlinear connected differential equations, considering the forces and moments acting on the UAV [7], [8]. The principle of flight simulation is a mathematical description using a nonlinear equation of motion. This can be examined in terms of stability and flight characteristics of the aircraft. Mathematical model also detailed explains the behavior of UAVs much more accurately but requires more computer resources.

#### 3.1. UAV with fixed wings aerodynamics

During the flight, forces are applied to the aircraft: lift, thrust, aerodynamic drag and gravity. The interaction of which affects the movement and position of the aircraft in space. The calculation of aerodynamic forces is described by formulas:

$$F_{x\_aero} = -((C_{xa} \cdot \alpha) + (C_{x0}) \cdot W_{surf} \cdot q_{bar}) \quad (1)$$

$$F_{y\_aero} = \left( \beta \cdot c_{y\beta} + \left( \frac{c_{yr} \cdot r \cdot W_{span}}{2 \cdot W_{aero}} \right) + c_{y\Delta RUD} \cdot \Delta RUD \right) \cdot W_{surf} \cdot q_{bar} \quad (2)$$

$$F_{z\_aero} = \left( C_{z0} + c_{za} \cdot \alpha + \left( \frac{c_{zq} \cdot q \cdot W_c}{W_{aero}} \right) + c_{z\Delta EL} \cdot \Delta EL \right) \cdot W_{surf} \cdot q_{bar} \quad (3)$$

Where:  $C$  - aerodynamics coefficients,  $W_{surf}$ ,  $W_{span}$ ,  $W_c$  - geometric dimensions of the wings (area, span, middle chord),  $\alpha$  - angle of attack,  $q_{bar}$  - atmospheric pressure,  $V_{aero}$  - airspeed,  $\Delta EL/\Delta RUD$  - horizontal / vertical deviations.

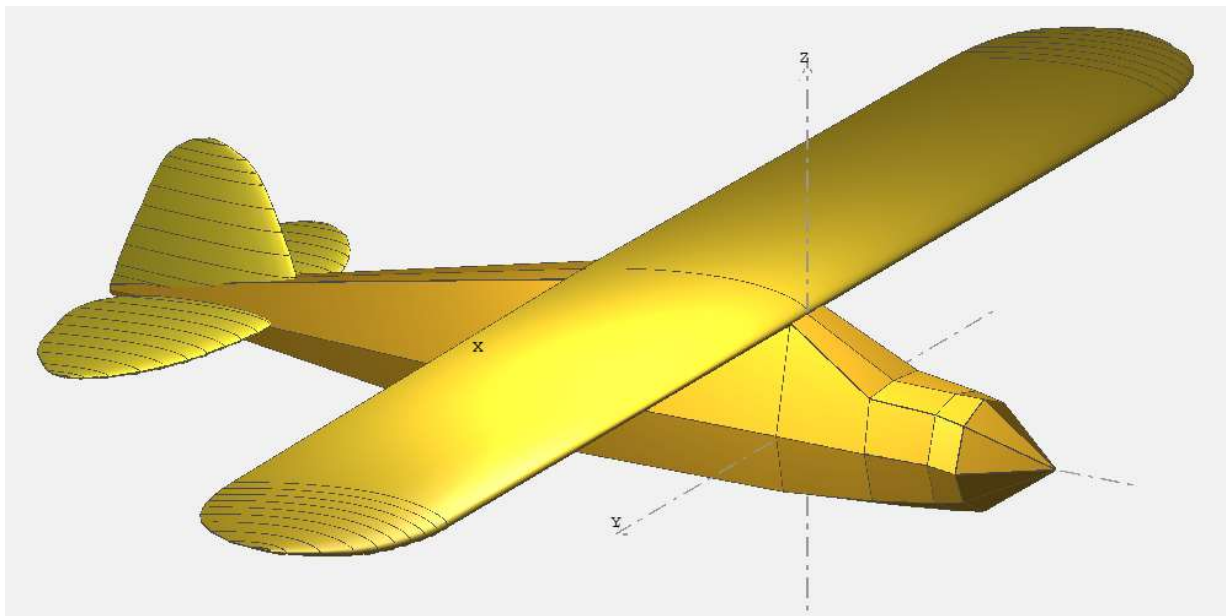
Aerodynamic moments are created by aerodynamic forces. These moments are described by formulas:

$$M_x = \left[ C_{t\Delta RUD} \cdot \Delta RUD + C_{t\Delta AIL} \cdot \Delta AIL + C_{t\beta} \cdot \beta + \left( \frac{C_{lp} \cdot p \cdot W_{span}}{2 \cdot V_{aero}} \right) \left( \frac{C_{lr} \cdot r \cdot W_{span}}{2 \cdot V_{aero}} \right) \right] \cdot W_{surf} \cdot W_{span} \cdot q_{bar} \quad (4)$$

$$M_y = \left[ C_{m\Delta EL} \cdot \Delta EL + C_{m\alpha} \cdot \alpha + C_{m0} + \left( \frac{C_{mq} \cdot q \cdot W_c}{V_{aero}} \right) \right] \cdot W_{surf} \cdot W_{span} \cdot q_{bar} \quad (5)$$

$$M_z = \left[ C_{n\Delta AIL} \cdot \Delta AIL + C_{n\Delta RUD} \cdot \Delta RUD + C_{n\beta} \cdot \beta + \left( \frac{C_{np} \cdot p \cdot W_{span}}{2 \cdot V_{aero}} \right) + \left( \frac{C_{nr} \cdot r \cdot W_{span}}{2 \cdot V_{aero}} \right) \right] \cdot W_{surf} \cdot W_{span} \cdot q_{bar} \quad (6)$$

Aerodynamic forces and moments are determined based on the geometry of the aircraft using tables, simulations, and wind tunnel experiments [9]. To create a mathematical model, it is necessary to know the distribution of pressure coefficients around the aircraft. This technique is combined with aerodynamic analysis performed using XFLR5 software [10]. The program uses the panel method in calculations and based on it can calculate the aerodynamic forces acting on the aircraft in accordance with its geometric dimensions. The program allows simulation to derive lift coefficients, which are used to create a mathematical model.



**Figure 2** UAV model created in XFLR5 system

### 3.2. Mathematical model of UAV

The 6DoF mathematical model in the simulation environment, shown in Fig. 2, is created based on the real parameters of the control object. The parameters indicate the aerodynamic properties and characteristics, the aerodynamic profile of the UAV and the relationships between the coordinates of the small UAV. It is a simulation platform for current and future developed flight control and flight trajectory. The input variables 6DoF model, which can be controlled manually according to a predefined signal or by the pilot-operator from the transmitter, are the deviations of the control surfaces (elevator, rudder, ailerons) and the thrust of the electromotor. The output quantities of the model represent the position angles and position of the UAV in space and their derivatives, aerometric velocity, forces acting on the UAV and moments.

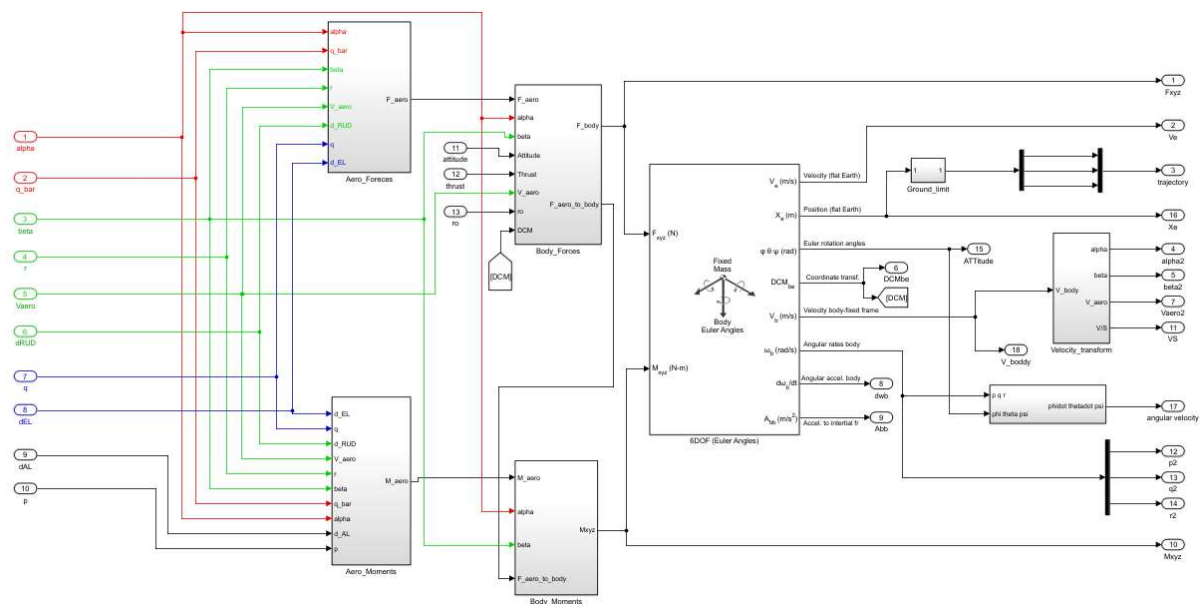


Figure 3 6DoF mathematical model in Matlab/Simulink

The created mathematical model serves as a test platform for verifying the correct operation of the system. The result of modeling is accuracy of the simulation. It shows all the necessary information about the behavior of the UAV, the magnitude of forces and moments that affect it, as well as its course of flight variables.

### 3.3. 6DoF UAV model verification

Verification of the mathematical 6DoF model UAV Carbon Cub 15cc performed in a simulation environment aims to ensure its accuracy and functionality. Functionality is verified by simulation analysis the changes elevator position. It's an initial test that gives the basic values of accuracy. Verification of the activity to control and stabilize the speed, height, position angles and their derivatives carried out by simulation experiments at specified parameters indicate its accuracy. All data used in the research give a representative sample to verify the complex motion of the aircraft. The outputs from the simulations verify functionality of the model.

Simulation testing analyzes the correctness of the change in longitudinal plane using the elevator in various changes of input parameters:

- Step command to change the position of the elevator at a constant speed of 100 km/h,
- A simple command using a signal builder to change the position of the elevator from the initial phase, at a constant speed of 100 km / h,

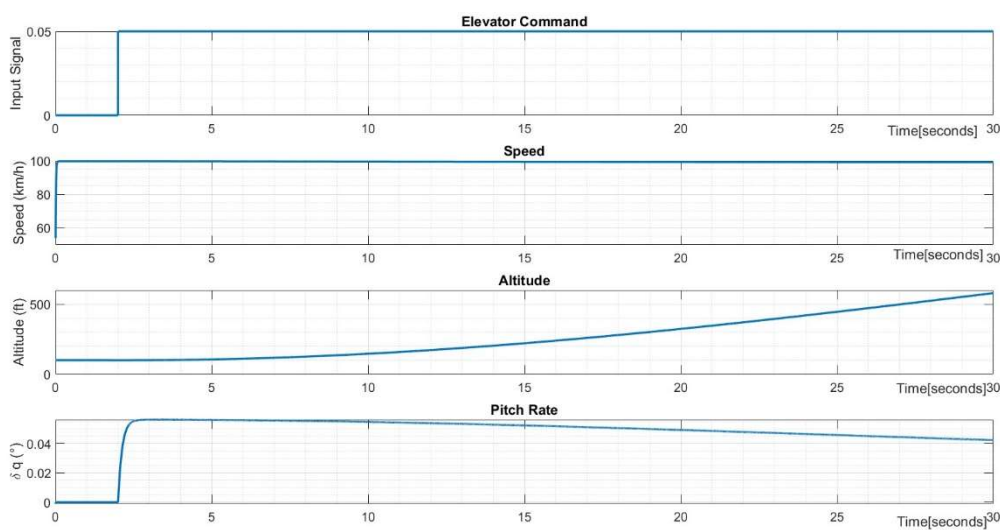
- Extended command using a signal builder to change the position of the elevator from the initial phase, at a constant speed of 100 km / h,
- Step command to change the position of the elevator at 75% of engine thrust.

**Table 2** Input command parameters to verify 6DoF

INPUT PARAMETERS				
FIGURE	ELEVATOR		SPEED/THRUST	
	Input method	Command value	Constand speed	Thrust
4.	step	[0.05] [-0.05]	100 km/h	x
5.	simple signal builder	[0.09] [-0.05] [-0.1] [-0.01][0]	100 km/h	x
6.	extended signal builder	[0.002] [0.08] [-0.08] [0.04][0]	100 km/h	x
7.	step	[0.05] [-0.05]	50/78/52/100 km/h	0.75 %

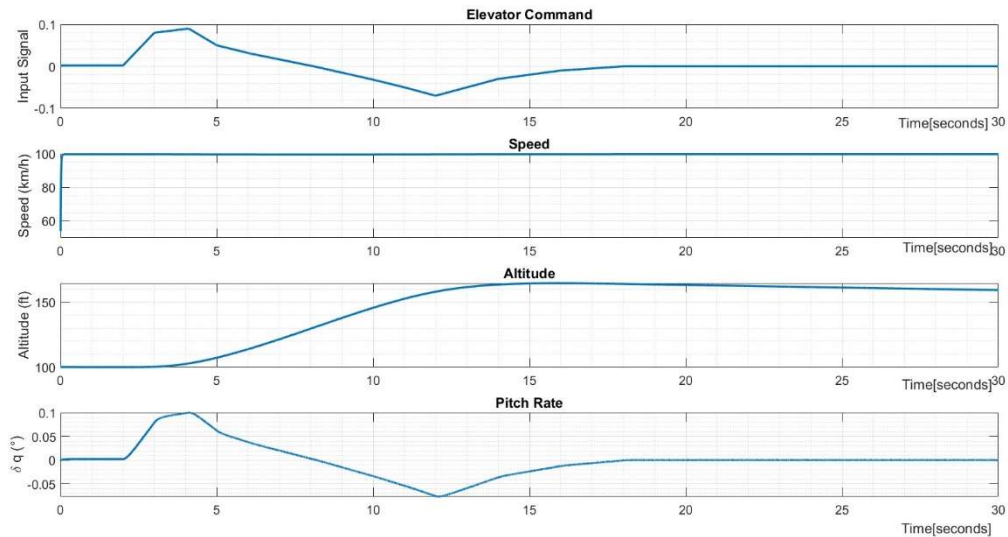
Tab. 2 briefly describes input parameters to verify 6DoF model with new coefficients. Step parameters show climb command to deviate elevator, and then return to elevator zero position (UAV stay in climbing position). To the next command use simple signal builder, where the UAV start climbing, then used negative value in signal builder to return UAV to stable horizontal position. The last input command is extended signal builder. Firstly, used positive value (UAV climbing), then used negative value for 6 seconds (UAV descending), and again used positive value 0.04 and value 0 to return UAV to the horizontal flight. Verification and functionality of the 6DoF model was performed at the automatic thrust, which was set at a speed of 100 km/h. Further testing took place at a command of 75% of the electro motor power. In both cases, with different parameters, the proposed 6DoF model works, as the UAV kept the speed at a constant thrust.

Fig. 4 shows the change in elevator, with different elevator commands. These commands were input parameter and the step, which was set according to the graph. The step value represented the elevator deviation from the initial phase and increase in the elevator angle by 0.05 °. Subsequently, in 9 seconds, the angle of attack in the longitudinal plane leveled and eased. For this situation to occur, the value had to be negative.



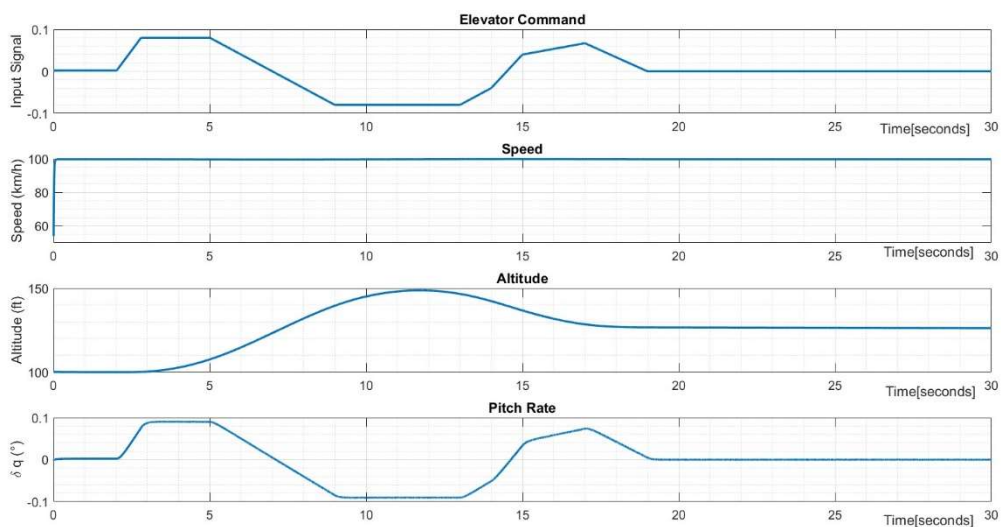
**Figure 4** Step input to 6DoF model in constant 100 km/h speed

Fig. 5 shows the change in elevator, height, and angle in the longitudinal plane at the input of a simple command. This consists in the requirement to deflect the elevator at constant speed of 100 km/h, and then to level the elevator. Observe this change, on the change in altitude, which shows the climb by a certain value and the subsequent alignment to level flight.



**Figure 5** Simple signal input to 6DoF model in constant 100 km/h speed

Fig. 6 presents the simulation extended by additional elevator commands. At the beginning of the simulation, there is an ascent, descent, and finally the UAV model. The figure proves the correct operation, where the commands for the elevator, speed, change of altitude and change of the angle of attack are shown.



**Figure 6** Extended signal input to 6DoF model in constant 100 km/h speed

The mathematical model was also verified at constant power, where the speed of the UAV changes with respect to the height. Shown in Fig. 7. The thrust was set at 75% and the graph shows that when

the height changes, the speed of the model also changes, which proves the correct functionality of the model and the elevator control itself.

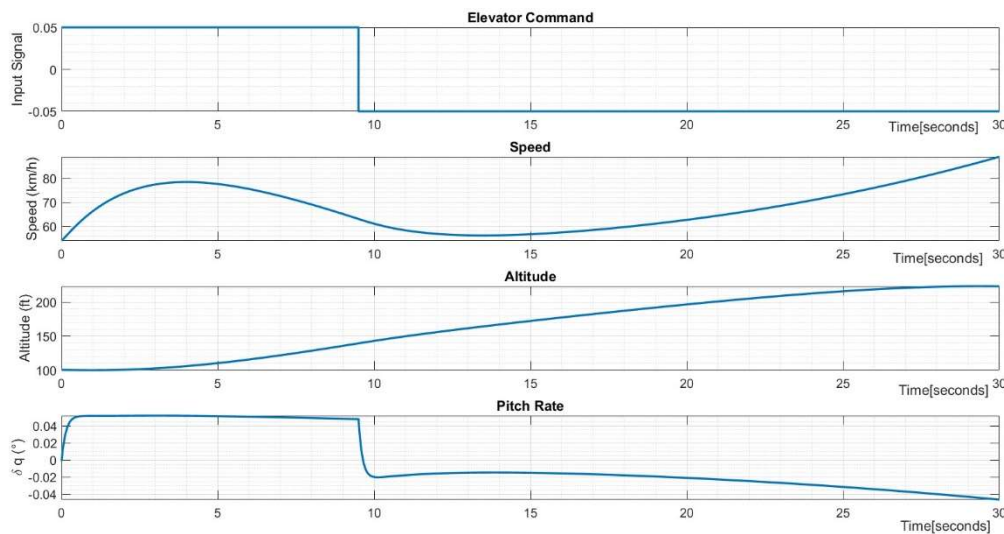


Figure 7 Step input to 6DoF model at 75 % thrust power

#### 4. CONCLUSION

The analysis of aerodynamic forces using the program XFLR5 and the created dynamic mathematical model of the aircraft in the Matlab / Simulink environment, described in the article, shows its ability to simulate the basic situational conditions of the aircraft. The article deals with the creation and functionality of a mathematical model found by simulation verification. Experimental verification explains the behavior of UAVs in basic mode in a simulation environment. The model thus helps to know the stability and maneuverability of a real UAV. Its use is possible for verification, and testing of advanced intelligent algorithms of integrated systems, which can then be implemented into a real UAV.

The challenge for the further studies could be to verify the ability of the mathematical model even in situational decomposition. The ability and functionality of the mathematical model in the critical situational states of the UAV and in individual situational classes would be ascertained.

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