ALGORITHMS IN AERODYNAMICS

Peter Szabó - Ľubica Čontofalská

The world's leading universities use a variety of books and electronic publications during courses focused on informatics, namely the algorithms and data structures. The article introduces the most famous publications, which are used at technically and mathematically aimed universities. On this basis, it is possible to design general classification of the most frequently used algorithms. In this classification algorithms in aerodynamics are also included. The work provides some examples of simple such algorithms. Later introduces a relatively new field of aerodynamics CFD (Computational Fluid Dynamics) – advanced technology with complex calculation methods used for fluid flow simulations.

K e y w o r d s: algorithm, classification of algorithms, aerodynamics, Computational Fluid Dynamics

1 INTRODUCTION

First types of algorithms were represented by mathematical calculation longer ages ago. Euclid's algorithm for finding the greatest common divisor (known for more than 2 300 years) is the proof. Amount and diversity of the data are rapidly growing continuously. So these days, computer algorithms working on big data are in center of attention. Billions of people around the world use computers daily while surfing the Internet, sending e-mails, or writing documents. Behind this every single activity, behind all computer technologies are algorithms. Finally, there are creators who inventing algorithms. Knowledge of algorithms and data structures theory is therefore required for novice and experienced programmers, as well as for students of computer science and other related fields. The world's leading universities such as Harvard, Yale, or Princeton, use next publications:

- The Art of Computer Programming (Donald E. Knuth)
- Introduction To Algorithms (Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein)
- Algorithms (Robert Sedgewick)
- Algorithms (Sanjoy Dasgupta, Christos H. Papadimitriou, Umesh V. Vazirani)

All these publications are written in a manner aimed at readers. It means that teachers, students, programmers and other professionals are able to understand. They are also intended for lay people interested in this area (e.g. self study). The publications provide important theoretical basis of algorithms and data structures. Also contain lot of pictures and illustrations which are helpful in explaining the way how the individual algorithms are working. Different kinds of exercises, some of them written in pseudo-code, some as complete java programs, are included.

2 INTRODUCTION TO ALGORITHMS

2.1 What is an algorithm?

The exact definition of an algorithm does not exist. Simply it can be defined as a set of instructions for solving the problem. Algorithms can be implemented or simulated by a computer program. They are necessary for computer processing. Application is set of algorithms used for solving specific task. Algorithm written in the programming language serves as communication tool between the programmer and the hardware. It has to meet following features:

- Elementariness. It consists of a finite number of simple and manageable actions - steps.
- Determinism. After every step, it is possible to determine whether the observed process was finished, and if not, next step has to be clearly identified for continuing.
- Finiteness. Number of repetitions of algorithm steps has to be finite. It means, that after the final number of steps, the algorithm must finish.
- Output. Solving the problem with help of the algorithm leads to the correct result - output.
- Collectiveness. It is possible to use the same algorithm for solving a large group of similar problems differing in inputs. [1] [2]

Simple interpretation of Euclid's algorithm: Two positive integers m and n are given. Find their greatest common divisor, i.e. the largest positive integer divisible by m and n.
E1. (Find remainder). Divide m by n, let r be the remainder (r will be in the range $0 \leq r < n$).
E2. (Is it equal to zero?). If $r = 0$, this algorithm ends and the result is n.
E3. (Interchange). Value of n assign into variable m ($m \leftarrow n$), Into variable n assign value of r ($n \leftarrow r$), continue with step E1. [3]

2.2 Classification of algorithms

There are many different ways of algorithms classification. According to the Algorithms publication by

Robert Sedgewick, best known algorithms are classified as follows:

- Sorting algorithms

Data sorting plays a major role in commercial data processing and in modern scientific calculations. It is used in transaction processing, combinatorial optimization, astrophysics, molecular dynamics, linguistics, weather forecasts, and many other areas.

- Searching algorithms

Internet, computer networks and informational technologies generally provide an immense amount of information. Without searching algorithms would not have been possible development of an informational infrastructure with which we are in touch daily.

- Graphs

Interconnections between entities, objects play an important role in many computational applications. Links represents relationship between objects. Questions arise: Is there a connection from one entity to another by following individual links? How many other entities are associated with the entity? What is the shortest existing connection between any two entities? Abstract mathematical objects - graphs are describing or modeling such states.

- String algorithms

Working with character strings of different lengths is very common in programming. String algorithms are mainly used in information processing, because almost all the information is encoded as text strings. Usage of these algorithms we can found in Genetics (genetic code is character string consisting of letters A, C, T and G), or communication systems (text messages sending requires text processing) as well as in many other information processing. [4]

3 ALGORITHMS IN AERODYNAMICS

Algorithms used in disciplines such as physics, medicine, bioinformatics, and aerodynamics are included into group of theory calculation algorithms. But this classification is very general since during development of algorithm for some discipline, the knowledge of several areas is required. Fluid mechanics, aerospace engineering, mathematics and computer science are necessary when developing aerodynamics algorithms. The multidisciplinary character of aerodynamics algorithms is shown in figure (Figure 1). The knowledge of all these areas was used during the development of aircrafts made by Boeing as well as other manufacturers.

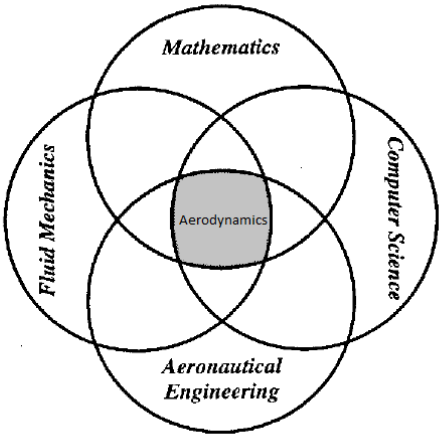

FIGURE 1: MULTIDISCIPLINARY NATURE OF ALGORITHMS IN AERODYNAMICS [5]

Students of Faculty of aeronautics are facing different fluid flow computations during their study. Some of the basic relationships and physical laws, used in these computations, are described by continuity equation, Bernoulli's equation, equation for determining the measure of air compressibility (Mach number) and equilibrium flight computation. Based on these relationships and knowledge of the basic principles and necessary physical laws it is possible to design an appropriate algorithms for solving the issue – propose an adequate computation method.

3.1 Procedure for computing the velocity in the tube at low speeds

Mass conservation law is used for the velocity in the streamtube computation. It is expressed by the continuity equation. According to the law, the amount of mass that enters the action is equal to the amount of mass exiting from the process. [6]

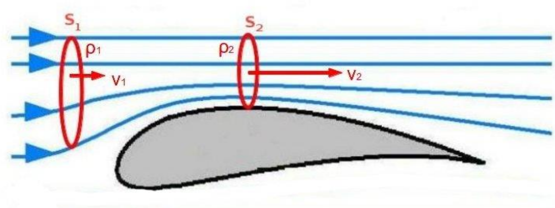

FIGURE 2: STREAMTUBE OVER THE AIRFOIL [7]

The figure (Figure 2) shows the streamtube of variable cross-section, marked S1 and S2 are areas of cross-sections. According to the law of conservation of mass, the same amount of air must flow through each tube place in a given time interval. Thus, over the section S1 flows the same amount of air as through the cross-section S2. [6]

The air compressibility must be considered during computations. It is depended on the flow velocity.

At low flow velocities (*v<360 km/h, M<0,3M*) we can think of the air as incompressible, there is no change of density (ρ = *constant*). The general equation of continuity for incompressible fluid has the form:

$$v_1 S_1 = v_2 S_2 \qquad (1)$$

where v_1 – airflow speed in the cross-section 1, S_1 – area of cross-section 1, v_2 – airflow speed in the cross-section 2, S_2 – area of cross-section 2. [8]

The general continuity equation leads to the conclusion that the velocity of airflow decreases if area of streamtube cross-section increases and opposite, if area decreases, the air flow velocity increases.

Problem formulation: Calculate the airflow velocity in cross-section S_1 of the streamtube, if area of S_1, S_2, and flow velocity v_2 are given.
Problem analysis: The formula for computing the air flow velocity v_1 is based on the continuity equation (1) and it looks like this:

$$v_1 = \frac{v_2 S_2}{S_1} \qquad (2)$$

Desired output: v_1 - airflow velocity in cross-section 1

Inputs: S1 – area of cross-section 1
S2 – area of cross-section 2
v2 – airflow velocity in cross-section 2

Steps:
[1] Input: S1, S2, v2
[2] Computation: v1 = (v2*S2) / S1
[3] Output: v1

3.2 Indicated and true airspeed computation method

For computing airspeed, the relation between static and dynamic pressure plays role in aviation aerodynamics. This relation is known as Bernoulli's equation, which expresses the conservation of energy law in the steady air flow. According to it, the amount of energy on input is equal as the amount of energy on the output. Thus, the sum of the static and dynamic pressure (total pressure) is constant. [6]

The general form of Bernoulli's equation:

$$p + \frac{\rho v^2}{2} = p_{total} = const., \qquad (3)$$

where p is static pressure and $\frac{\rho v^2}{2} = q$ is dynamic pressure.

According to Bernoulli's equation for incompressible medium it is true: if speed increases, similarly, dynamic pressure is increasing and static pressure is decreasing. Opposite to this, reduction in speed leads to the dynamic pressure decreases and static pressure increases. This sets out that it is possible to determine the flight speed according to the value of dynamic pressure. At the point where the speed is increased (and thus also the dynamic pressure is increased), static pressure is reduced for keeping constant value of the total pressure. [6]

Problem formulation: Compute the flight speed if pilot-static tube sensor indicates p_{total} for total pressure and p_{stat} for static pressure during the flight. By using a value of ρ determine, if the flight speed is indicated or true airspeed.

Problem analysis: The formula for calculating the indicated and true airspeed was derived of the Bernoulli's equation (3) and looks like this:

$$v = \sqrt{\frac{2(p_{total} - p_{stat})}{\rho}}, \quad (4)$$

Desired output: v – flight speed

Inputs: ρ – air density at a given height
p_total – total pressure
p_stat – static pressure

Steps:
[1] Input: ρ, p_total, p_stat
[2] Computation: v = sqrt(2*(p_total – p_stat) /ρ)
[3] If ρ = 1,225 output is: INDICATED SPEED "v"
else output is: TRUE AIRSPEED "v"
[4] End of algorithm

3.3 Mach number computational algorithm

The speed of sound waves spreading air mass in a certain place or in a particular area is called the local speed of sound. It depends on the absolute temperature of the air at a given altitude. As the air temperature decreases on higher altitudes, therefore the sound waves are spreading the air slower, the higher the plane flies. Relation for determining the speed of sound is as follows:

$$a = 20{,}05\sqrt{T}, \quad (5)$$

where a – speed of sound in the air, T – absolute temperature of the gas. [8]

Mach number is a measure of the air compressibility and it is the ratio between the true airspeed (TAS) and the local speed of sound.

$$M = \frac{v_{TAS}}{a}, \qquad (6)$$

The sound travels through the standard atmosphere at speeds of 340 $m.s^{-1}$ (1228 $km.h^{-1}$) at zero altitude, gradually decreases and stops at the value of 296 $m.s^{-1}$ (1065 $km.h^{-1}$) at 11 000 m altitude. [8]

Problem formulation: Calculate the value shown on aircraft's machmeter, if the true airspeed v_{TAS} and absolute temperature at a given altitude T are given.

Problem analysis: From equation (5) we determine the local speed of sound a. Using the definition of Mach number (6) we determine the true airspeed.

Desired output: v_TAS – true airspeed

Inputs: T – absolute temperature
M – Mach number

Steps:
[1] Provide inputs: T, M
[2] If T is given in Celsius degrees (C) then T = C + 273,15, else continue with step [3]
[3] Calculate local airspeed: a = 20,05.sqrt(T)
[4] Calculate true airspeed: v_TAS = M*a
[5] Output: v_TAS

3.4 Minimum speed for equilibrium flight computation

Equilibrium flight exists when the aircraft flies at a constant altitude without acceleration (constant velocity) (Fig. 3). Such stable flight occurs in case when the sum of all forces and the sum of all moments around the aircraft's center of gravity are equal to zero. During the horizontal stable flight the following forces acts on the plane:

- lift (L)
- drag (D)
- thrust (T)
- weight (W) [9]

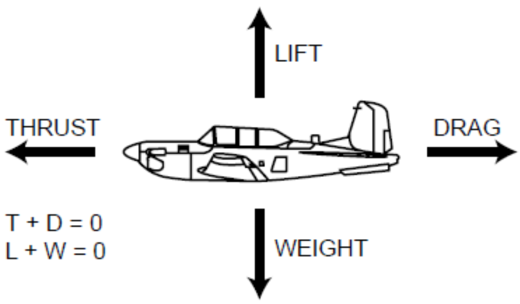

FIGURE 1: EQUILIBRIUM FLIGHT [9]

The balance of forces is simply described as follows:

$$W + L = 0, \qquad (7)$$
$$D + T = 0, \qquad (8)$$

The above equations show, that these forces cancel each other, if thrust is equal to drag and lift is equal to weight. Under these conditions, the acceleration is zero, the aircraft is in steady flight mode.

We can express equation (7) in the form:

$$W = c_y \frac{\rho v^2}{2} S, \qquad (9)$$

and then determine the relationship between flight speed and lift coefficient c_y:

$$v = \sqrt{\frac{2W}{\rho S c_y}} = \sqrt{\frac{2mg}{\rho S c_y}}, \qquad (10)$$

where W – weight of aircraft, c_y – lift coefficient, S – planform area, m – mass of the aircraft, g – acceleration due to gravity ($g = const. = 9{,}806\ m.s^{-2}$).

The above equation (10) shows, that the speed decreases when the lift coefficient increases. [8]

Problem formulation: Calculate the minimum speed for aircraft with a mass m, the air density ρ, wing area S and maximum lift coefficient $c_{y\,max}$.

Problem analysis: From relation (7) between flight speed and lift coefficient we determine minimum speed v_{min} for equilibrium flight.

$$v_{\min} = \sqrt{\frac{2mg}{\rho S c_{y_{\max}}}}, \qquad (11)$$

where v_{min} – minimum speed, $c_{y\ max}$ – maximum lift coefficient.

Desired output: v_min – minimum speed for equilibrium flight

Inputs: m – mass of the aircraft
g – acceleration due to gravity
(g = const. = 9,806 $m.s^{-2}$)
ρ – the air density
S – wing area
cy_max – maximum lift coefficient

Steps:
[1] Provide inputs: m, g, ρ, S, cy_max
[2] Calculate minimum speed:
v_min = sqrt((2*m*g)/(ρ*S*cy_max))
[3] Output: v_min

4 COMPUTATIONAL FLUID DYNAMICS

Computational Fluid Dynamics (CFD) is a relatively new field of aerodynamics. It is used to simulate the physical phenomena and aerodynamic effects by using specialized software and supercomputers. Computer simulation of flow is realized on the basis of mathematical models using numerical methods, computational algorithms and special programs. The result is realistic qualitative and quantitative flow visualization. Results obtained with CFD are comparable with results obtained from experiments, e.g. from wind tunnels. [10]

Kinds of CFD calculations:

- 2D and 3D flow
- internal and external aerodynamics
- stationary and non-stationary analysis
- compressible, incompressible, subsonic, supersonic flow
- laminar and turbulent flow
- multiphase flow
- optimizing the aerodynamics

4.1 Aviation use of CFD

The world's most famous aircraft manufacturers such as the Boeing and Airbus, as well as many others, use CFD to improve and optimize the old and new products. This technology is currently used in the optimization of all components that are affected by fluid flow. Examples include the prediction of performance, design of high-speed wings, avionics ventilation, icing forecast, fuel system design, and aeroacoustics for noise reduction. The major issues that CFD deal with are aerodynamic design of overall aircraft construction and wing design. [5]

FIGURE 4: CFD SIMULATION OF AEROACOUSTICS [5]

4.2 Optimizing airfoil

Airfoil design research focuses on such a wing construction, which is resistant to vibration at high speeds. Supersonic speeds generate shock waves causing turbulence and strain construction. Using CFD by optimizing the airfoil design can be found shock free airfoil in less than one minute. [5]

Figure 5 illustrates the simulation of compressible fluid flow around the airfoil NACA0012. With this simulation, we can observe the distribution of Mach number M around the airfoil at a velocity speed twice as large as speed of sound (M = 2). Shock waves occurring in supersonic environment represent sharp gradients. [12]

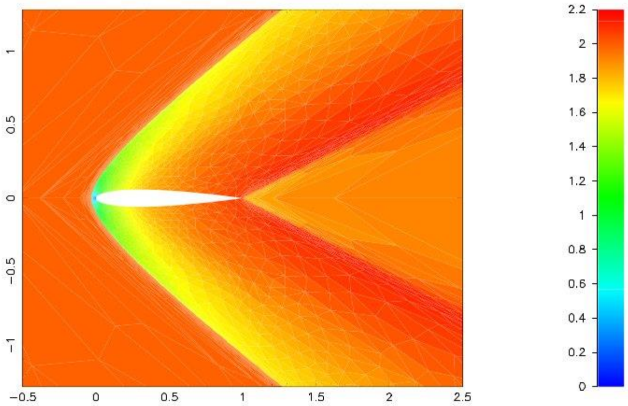

FIGURE 5: AIRFOIL VISUALIZATION

5 CONCLUSION

This article highlights the need to study the theory of algorithms, as they are integral part of all Information Technology. Mentioned publications deal with design and analysis of algorithms and data structures. They serve as textbooks used during courses focused on computer science at the world's leading universities:

- The Art of Computer Programming (Donald E. Knuth)
- Introduction To Algorithms (Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein)
- Algorithms (Robert Sedgewick)
- Algorithms (Sanjoy Dasgupta, Christos H. Papadimitriou, Umesh V. Vazirani)

According to the Algorithms publication by Robert Sedgewick, best known algorithms are classified as follows:

- Sorting algorithms
- Searching algorithms
- Graphs
- String algorithms

This text contains 4 examples of algorithms in aerodynamics. They are inspired by the examples of aerodynamics and fluid mechanics, to which students of Faculty of aeronautics are facing. In these computations, basic relationships and physical laws as continuity equation, Bernoulli's equation, equation for determining the measure of air compressibility (Mach number) and equilibrium flight computation are used. Based on these relationships and knowledge of the basic principles and necessary physical laws it is possible to design an appropriate algorithms for solving the issue – propose an adequate computation method. Article also describes CFD - computer technology that uses numerical calculations of fluid mechanics equations. In aviation, the key role of CFD simulation is in the design of overall aircraft structure and wing design. Aircraft manufacturers such as Boeing and Airbus, also use CFD simulation to optimize the different components affected by airflow.

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AUTHORS' ADDRESSES

Ľubica Čontofalská, Ing., Faculty of Aeronautics - Technical University of Košice, Department of Air Traffic Management, lubica.contofalska@tuke.sk

Peter Szabó, RNDr., PhD., Faculty of Aeronautics - Technical University of Košice, Department of Aerodynamics and Simulations, peter.szabo@tuke.sk