MATHEMATICAL MODELS OF CHANNEL AND ACTUAL SPED OF MACH

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The aim of this article is to explain simulation model with focus on computing flight parameters such as flight height and speed in the programming environment of MATLAB to reader. The article is based on analytical, mathematical and simulation principles. The problem of atmosphere pressure with correspondence to the international standard atmosphere, flight height and speed simulation computing, Mach´s number computing and the dilemma of the impact of angle of attack and angle of yaw on the speed of flight is solved in the article. The effect of interchanging real and standard static temperature on computing the flight height as well as the matter of air pressure, air temperature and air density changes inflicted by weather alternation are considered in the article.

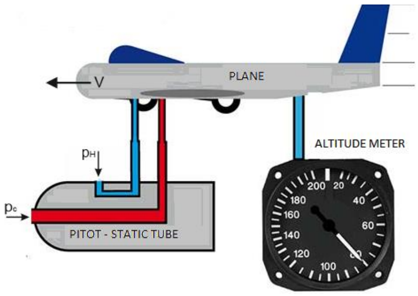
K e y w o r d s: pressure, altitude, speed, Mach´s number, angle of attack, angle of yaw

1 INTRODUCTION

One of the tasks in plane navigation a piloting is positioning and control of plane´s movement in the atmosphere which demands a cluster of flight instruments called instruments for flight control. These instruments can be sorted into few categories. One contains flight instruments indicating advance motion of the plane´s centre (altitude meter, speed meter), rotational movement around the plane´s centre (gyro horizont, turn indicator), acceleration and aerodynamics' parameters (accelerometer, Machmeter) [1, 2]. Without these, flying in zero visibility, representing flying in night, fog and clouds is impossible. Thus flight´s security depends on reliability of these. The analysis of altitude meter, which informs the crew about current flight height, is inducted in detail in the article. Altitude meter is an instrument that evaluates the height based on changes in static air temperature and static air pressure. Simulation model for altitude meter solves the problem of interchanging real static temperature with standard static temperature and alternation inducted by weather changes when the static pressure, total pressure, temperature and temperature gradient change. Later on, the analysis of altitmeters placed in cockpit for crew´s information about the speed of the plane´s movement in the atmosphere is described. Simulation model for altitude meter solves the problems arising from the plane´s flight in incompressible and compressible air. The dynamic pressure, the main informant on speed, changes in the altitude meter based on air compression. With flight speed, Mach´s number, which is expressed by flight speed to sound speed ratio, is aligned. Thus the simulation model solves computing Mach´s number depending up speeds lower than 1M and speeds higher than 1M.

2 MATHEMATICAL DESCRIPTION OF BAROMETRIC ALTIMETER

Barometric altimeter, which only has one pressure port, is used for measuring static pressure (see pic. 1).



Picture 1 Altitude meter connect to Pitot-static tube

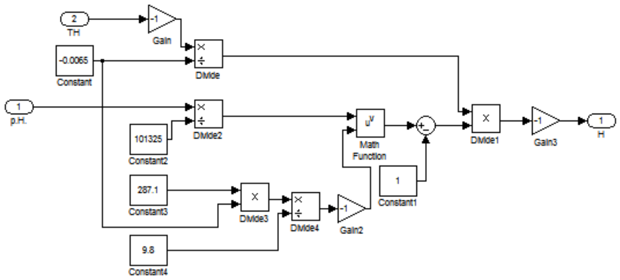
Then the ideal barometric altimeter is calibrated according to calibration equation (2.1):

$$H_{pr} = -\frac{T_0}{\alpha} \cdot \left[1 - \left(\frac{p_H}{p_0} \right)^{-\frac{R \cdot \alpha}{g}} \right] \qquad (2.1)$$

where:

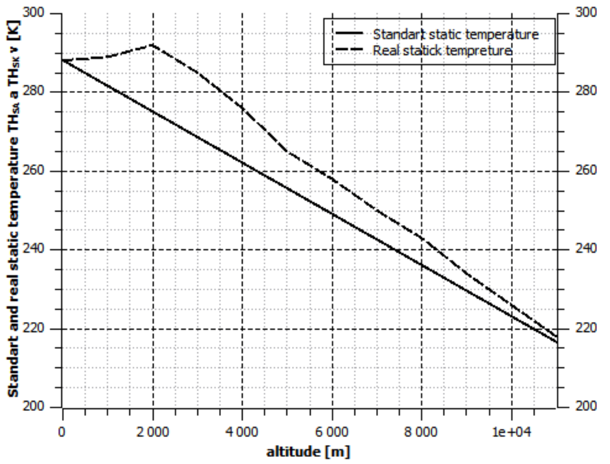
T_0- is temperature at the sea level
α - is temperature gradient
P_H- is static pressure in height
P_0- is pressure at the sea level
R- is gas constant
g- is mass acceleration

Simulation model (see pic. 2) was designed on basis of equation (2.1). Altitude meter computes either instrumental altitude, when zero temperature is instituted or total altitude, when static temperature is instituted.



Picture 2 Model of simulating for barometric altimeter

Instituting static air temperature into computing apparatus brings about certain error in the simulation model. Standard static temperature is temperature calculated according to ISA, but it differs from real static air temperature as depicted (see pic. 3).



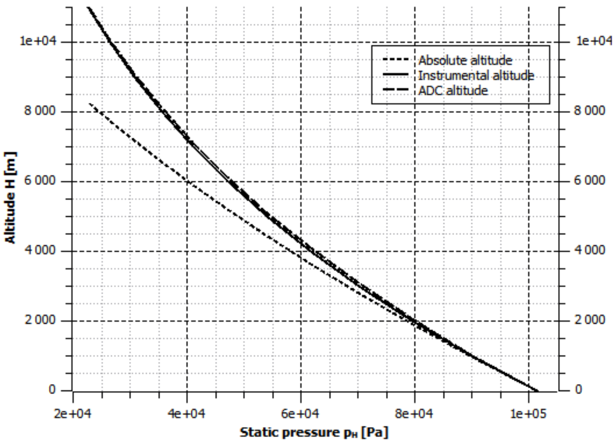
Picture 3 Graphic representation standart and real static tepmerature

In graphical depiction (see pic. 4) altitude deviation between barometric altimeter and ADC is alleged. ADC computes with equation (2.2):

$$H = -\frac{R}{g}\cdot\left(T_H - T_0\right)\cdot\frac{\ln\frac{p_H}{p_0}}{\ln\frac{T_H}{T_0}} \qquad (2.2)$$

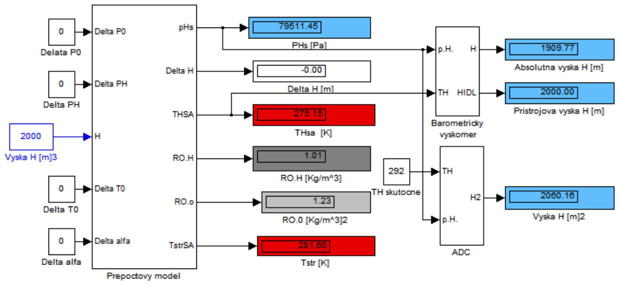
where:

R- is gas constant
g- is mass acceleration
T_{HSK}- is real static temperature
T_0- is temperature at sea level
P_H- is static pressure
P_0- is pressure at the sea level



Picture 4 Graphic representation altitude of barometrick altimeter and ADC

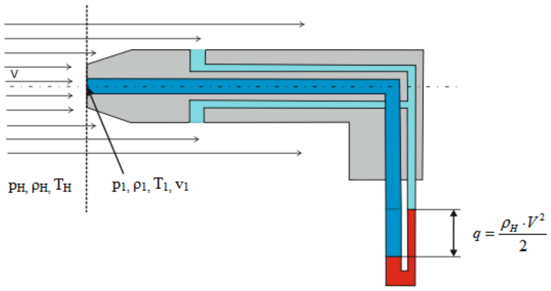
Altitude H is an input into model of calculation (see pic. 5) because the calculation is designed to recalculate altitude set by us into values of static pressure, absolute altitude errors, standard static temperature, air density, air density at sea level and average air layer temperature according to ISA.



Picture 5 Model of barometric altimeter

3 SPEED MEASURING METHOD

Plane´s speed is determined from difference of pressures which originate in Pitot-static tube wrapped by air (see pic 6). In the front opening of the Pitot-static tube , total pressure is formed. Openings along cylindrical surface of Pitot-static tube are called static openings and are used for measuring of static pressure p_H. To determine plane´s speed, dynamic pressure q, originating in difference between total pressure p_c and static pressure p_h, is used [4].



Picture 6 Pitot-static tube and formation dynamic pressure

Types of respective speeds used in aviature:

- Instrumental speed- is uncorrected entry from altitude meter built-in in the plane.
- IAS indicated speed- is instrumental speed corrected for instrumental error given by instrument´s construction.
- CAS calibrated speer- is calibrated speed corrected for positional error of Pitot´s tube.
- EAS equivalent speed- is calibrated speed corrected for air compressibility factor
- TAS true air speed- is plane´s speed relative to surrounding atmosphere.

TAS is thus equivalent speed corrected for altitude impact.

When computing speed, we will handle ideal speedmeter for which total and static pressure are measured in ideal Pitot - static tube, without deforming measured pressures,

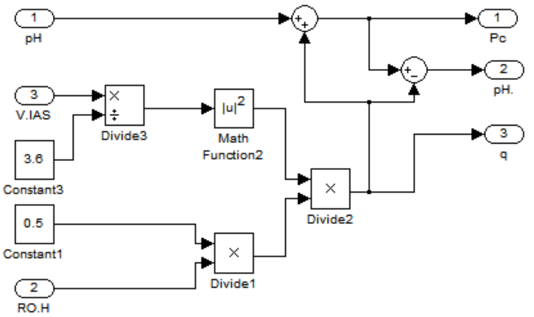
at steady plane´s movement in incompressible atmosphere.
The difference of pressures in incompressible atmosphere which effects the altitude meter (see pic. 6) is expressed as (3.1), [4]:

$$p_c - p_H = \frac{\rho_H \cdot V^2}{2} = q \qquad (3.1)$$

where:

- p_c- is total pressure
- ρ_H- is specific air density
- v- is speed
- q- is dynamic pressure.

Simulation model (see pic. 7) for computing dynamic pressure was designed on basis of below listed equation (3.1).



Picture 7 Model of simulating for dynamic pressure

As results from the last equation, plane´s speed depends also on specific air density ρ_H. After inducting sea level´s air density ρ_0 it is possible to create a ratio between the two densities. It means that if specific air density ρ_H is just equal to ρ_0, the speedmeter shows plane´s true speed V_{TAS}.
Thus it is needed to deduce which speed the speedmeter shows when the plane is moving in the atmosphere where the air density ρ_0 is different from the specific air density ρ_H. Speedmeter then shows indicated speed V_{IAS} which is calculated from coequality of dynamic pressure (3.2):

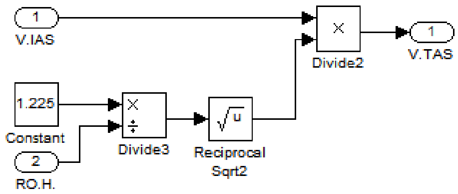
$$\frac{\rho_0 \cdot V_{IAS}^2}{2} = \frac{\rho_H \cdot V_{TAS}^2}{2} \qquad (3.2)$$

where:

- ρ_0 - is air density at the sea level
- V_{IAS} - is plane´s indicated speed
- ρ_H- is specific air density
- V_{TAS}- is plane´s true speed

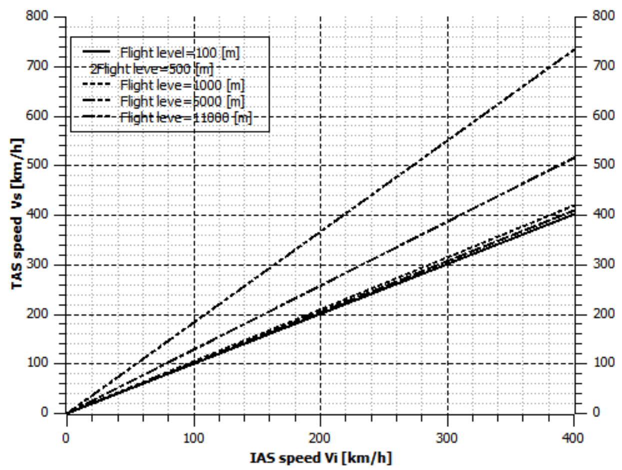
Simulation model (see pic. 8) of computing true airspeed was designed based on equation (3.3):

$$V_{TAS} = V_{IAS} \sqrt{\left(\frac{\rho_0}{\rho_H}\right)} \qquad (3.3)$$



Picture 8 Model of simulating for TAS speed

There is a change of the plane´s flight true airspeed as dependent on alternation indicated flight speed and flight level in which the plane flies, as shown in graphic representation (see pic. 9). The graph was calculated from the equation (3.3).



Picture 9 Graphic representation TAS speed and IAS speed in depending on flight level

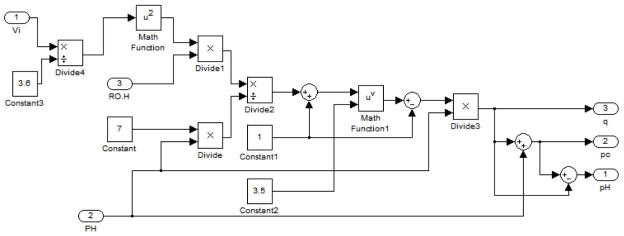
Above listed equations for ideal speedmeter were inducted provided the air is an incompressible medium. When deriving equations for speedmeter measuring speeds in subsonic zones, we have to start from knowledge of thermodynamics. Dynamic pressure for compressible medium then can be expressed, based on knowledge of thermodynamics and mathematically correct treatment, by equation (3.4):

$$q_c = p_{cc} - p_H = p_H \cdot \left[\left(1 + \frac{\rho_H \cdot V_{IAS}}{7 \cdot p_H} \right)^{3,5} - 1 \right] \qquad (3.4)$$

where:

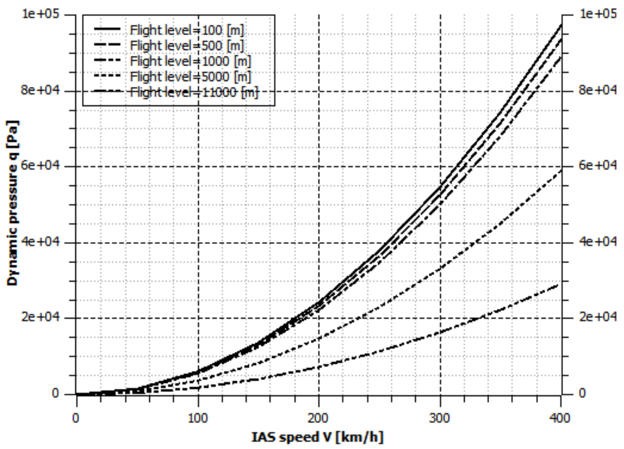
- p_{cc} is total pressure in compressible medium
- p_H is static pressure
- ρ_H is specific air density
- k is a constant, k=1,4
- v_{IAS}- is indicated speed

Simulation model (see pic. 10) for calculating dynamic pressure q_c in compressible medium was designed on basis of equation (3.4):



Picture 10 Model of simulating dynamic pressure in compresible medium

There is a change of the dynamic pressure in compressible medium as dependent on alternation of flight speed and flight level in which the plane flies, as shown in graphic representation (see pic. 11). The graph was calculated from the equation (3.4). Dynamic pressure figures were compared with the GOST norm.



Picture 11 Graphic representatoin dynamic pressre dependin on IAS speed

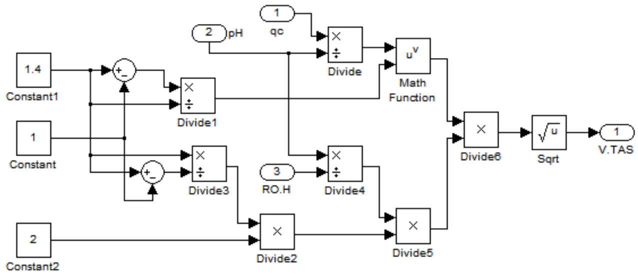
Calibration equation for computing speedmeter´s speed V_{TAS} in compressible medium can be obtained by explicitly expressing the speed V_{TAS} from above listed equations in the form: (3.5):

$$V_{TAS} = \sqrt{\left[\left(\frac{q_c}{p_H}+1\right)^{\frac{k-1}{k}} -1\right] \cdot 2 \cdot \frac{k}{k-1} \cdot \frac{p_H}{\rho_H}} \qquad (3.5)$$

where:

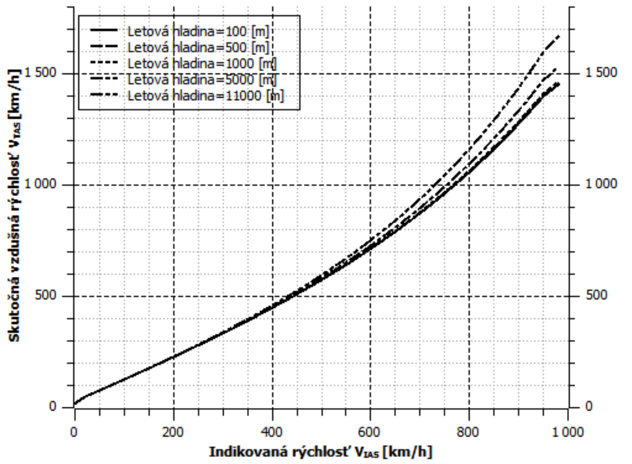
- q_c -is dynamic ram pressure
- p_H -is static pressure
- k -is a constant, k=1,4
- ρ_H -is specific air density
- ρ_0 -is air density at the sea level.

Simulation model for computing true airspeed (see pic. 12) was designed based on equation (3.5).



Pictre 12 Model of simulating TAS speed in compresible medium

There is a change of the true airspeed of flight as dependent on alternation of flight speed and flight level in which the plane flies, as shown in graphic representation (see pic. 13). The graph was calculated from the equation (3.5).



Picture 13 Graphic representation TAS speed and IAS speed in depending on flight level in compressible medium

4 MACH NUMBER CALCULATING METHOD

At high flight speeds, the aerodynamic characteristics of the aircraft may change rapidly due to the compressibility of air. Thus a sharp decrease in buoyancy occurs while resistance is increased, making aerodynamic forces decrease. The immediate effect of the magnitude of air compressibility can be characterized by Mach´s number. Crews judge aerodynamic characteristics of transonic and supersonic planes in high speed flight areas on basis of Mach´s number. The construction of Machmeter is similar to speedmeter´s construction with partial correction on density. The two instruments differ only in the construction of display section. Mach´s number is proportional expression of true airspeed due to the speed of sound waves- the speed of sound, which can be expressed as (4.1):

$$M = \frac{V_s}{a} = \frac{V_s}{\sqrt{k \cdot R \cdot T_H}} \qquad (4.1)$$

where:

p_{cc} -is total pressure in compressible medium
p_H -is static pressure
k -is a constant, k=1,4.

After inducting an expression (4.2) for true speed into Mach´s number expression:

$$V_s = V_i \sqrt{\left(\frac{\rho_0}{\rho_H}\right)} = \sqrt{\frac{2\cdot(p_c - p_H)}{\rho_H}}\ , \qquad \text{for } \rho_H = \rho_0 \cdot \left(\frac{T_H}{T_0}\right)^{-\left(1+\frac{g}{R\cdot\alpha}\right)} \tag{4.2}$$

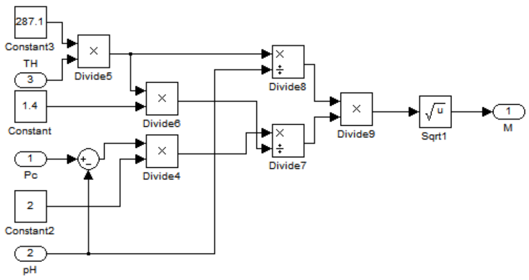
Then, by mathematically correct treatment, an equation for Mach´s number calculating in subsonic zones can be expressed as (4.3).

$$M = \sqrt{\frac{2\cdot(p_c - p_H)}{k\cdot R\cdot T_H}\cdot\frac{R\cdot T_H}{p_H}} \tag{4.3}$$

where:

p_{cc} -is total pressure in compressible medium
p_H -is static pressure
R -is gas constant
T_H -is static temperature
k -is a constant, k=1,4.

Simulation model for computing Mach´s number (see pic. 14) for subsonic zones was designed on basis of equation (4.3):



Picture 14 Model of simulating Mach’s number in sobsonic zone

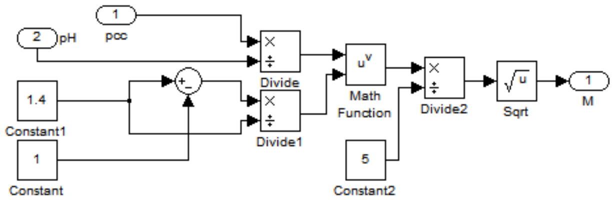
When plane is moving at supersonic speeds, perpendicular and oblique shock waves form around the Pitot tube. Perpendicular and oblique shock waves forming results in alternation in specific air density, thus equation for Mach´s number computing at supersonic speeds is used in form (4.4):

$$M = \sqrt{\left[\left(\frac{p_{cc}}{p_H}\right)^{\frac{k-1}{k}} - 1\right]\cdot\frac{2}{k-1}} \tag{4.4}$$

where:

p_{cc} -is total pressure in compressible medium
p_H -is static pressure
k -is a constant, k=1,4.

Simulation model for computing Mach´s number (see pic. 15) for supersonic speeds was designed on basis of equation (4.4).



Picture 15 Model of simulating Mach’s number in supersonic zone

Graphic representation (see pic 16) shows Mach´s number due to dynamic pressure. Thus in graphic representation, Mach´s number figure is compared in proportion due to total pressure and static pressure according to below mentioned equations (4.6), (4.7), (4.8):

- Mach´s number calculation for M<1 zones:

$$\frac{p_c}{p_H} = 1 + \frac{k}{2}\cdot M^2 \tag{4.5}$$

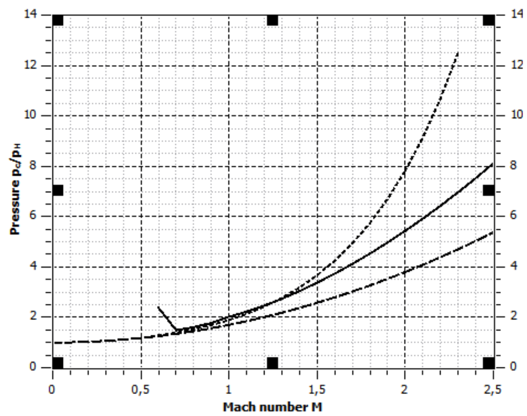
- Mach´s number calculation for M≥1 zones:

$$\frac{p_c}{p_H} = \frac{k+1}{2}\cdot M^2\cdot\left[\frac{(k+1)\cdot M^2}{4\cdot k\cdot M^2 - 2\cdot(k-1)}\right]^{\frac{1}{k-1}} \tag{4.6}$$

- Mach´s number calculation for M▯ 1zones:

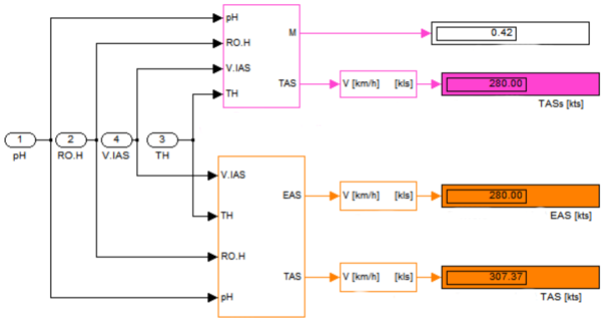
$$\frac{p_c}{p_H} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \tag{4.7}$$

Compressibility of medium is negligible up to Mach´s number zone M=0.6, as results from graphic representation (see pic. 16).



Picture 16 Graphic representation Mach number on pressure p_c and p_H

After combining respective simulation models, a subsystem for altitude meter and Machmeter is designed (see pic 17).



Picture 17 Model of speed meter

5 CONCLUSION

The simulation model was designed to evaluate the data from barometric altimeter, ideal speedometer in incompressible medium, speedometer with partial density correction for compressible medium and Machmeter. Input for calculation of the above mentioned data consists of conversion simulation model which, based on entered altitude, simulates necessary data corresponding with standard atmosphere ISA.

The designed simulation model of the ideal speedmeter canal in incompressible medium simulates real airspeed rate for incompressible environment. Correctness of simulated values for calculating the actual airspeed figure was verified only by the accuracy of dynamic pressure simulation values, which was compared with standard GOST table for various flight levels published on the Internet.

The difference between true airspeed data values and data for equivalent flight speed was proved for simulation canal for ideal speedometer in compressible medium, thus correctness of proposed simulation canals was proved.

The simulation canal for calculating Mach´s number was designed based on given theory. According to simulated values, it is obvious from graphic representation that Mach´s number value has a major role in assessing the compressibility of medium, thus also has an impact on true airspeed figure with which the crew would be informed.

ACKNOWLEDGMENT

This work was funded by the European Regional Development Fund under the Research & Development Operational Programme project entitled “Construction of a research & development laboratory for airborne antenna equipment, ITMS: 26220220130.”

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