GAS TURBINE COMPRESSOR DISC NATURAL FREQUENCY CALCULATION

Michal Horváth – Stanislav Fábry

The topic introduces theoretical work done in a diploma thesis. It contains procedure of calculating the natural frequency of gas turbine compressor or turbine rotor disc. Relations for calculating the natural frequency of a disc oscillation are step by step derived therein. To calculate the natural frequency is used Rayleigh's energetic method and the calculation for number of nodal lines is based on the specific procedure made in the MS Excel spreadsheet.

K e y w o r d s: frequency, oscillation, disc

1 INTRODUCTION

In the thesis, there were analyzed the causes and shape oscillation disc, natural frequency and manner of oscillation drives. The reason for this thesis was the lack of literature relevant to this issue in Slovakia.

Problems of construction and vibration of gas turbine engine discs is analyzed in particular depth in the study of aerospace engineering. The thesis could according to the depth of problem solving facilitate a comprehensive understanding of the issues and thereby assist in teaching and learning at the Faculty of Aeronautics at Technical University of Kosice.

An important part of the engine air – gas tract is rotating compressor and turbine. The different types of engines have different design solutions and different types of their rotors.

In an aircraft gas turbine is the most commonly used disc-drum design of compressor and turbine rotor. When rotating disc is stressed by centrifugal forces from its own weight and the weight of the blades, there are formed radial and tangential tensions. In addition, there is a turbine thermal tension from the temperature gradients. Loading forces and torques acting to the compressors and turbines have a cyclic character, causing vibration excitation.

Harmonic oscillation of a disc point **P** with its own angular frequency Ω, and the number of angular lines m can be described using the following equation:

$$w \equiv w(r, \varphi, t) = w(r)\, w(\varphi)\, w(t) = w(r) \sin(m.\varphi).\cos(\Omega.t) \tag{1}$$

Here w represents the instantaneous displacement of point **P** from the equilibrium in the z direction. Dimensional curve $w(\varphi) = \sin m \bullet \varphi$, respects change of amplitude of displacement only, depending on the angular position φ, while the curve $w(r)$ respects the change again only depending on the change of the radius. Curve $w(r)$ have to satisfy not only the boundary conditions of the disc montage, but it also has to respect the appropriate number of nodal circles.

2 RAYLEIGH METHOD OF DISC FREQUENCY CALCULATION

Suppose disc undamped harmonic oscillation with *"m"* nodes and angular frequency Ω according to Eq. (1). Provisionally and only due to simplicity we consider a permanent full disc thickness ($h = const.$) with radius R. Oscillations of any disc element dm or any part of the entire disc can be thought of as a process accompanied by adequate changes of kinetic and potential energy, although modifying their sum. This means that when there is the maximal potential energy of the oscillating disc must be the kinetic energy zero, and vice versa. For this condition occurs at the maximum and zero deflection disc element from the equilibrium position.

Natural frequency f of oscillating disc is then calculated from the condition of equality of maximal potential and kinetic energy as the speed and kinetic energy of disc vibration depend on the frequency.

Eg the maximum kinetic energy of half a circular sector of the disc between two nodal diameters, ie within the angles $\varphi = 0$ and $\varphi = \pi/2m$ apply:

$$K_{max} = \int_0^R \int_0^{\frac{\pi}{2m}} \frac{1}{2} dm.\vartheta_{max}^2 \tag{2}$$

Where: $dm = \rho\, h\, r\, dr\, d\,\varphi,\ \vartheta_{max} = \Omega\, w(r) \sin m\, \varphi$ (3)

Relation for maximum oscillation speed we get by derivation of equation (1) according to time, when there $\sin \Omega t = 1$

For K_{max} is then valid:

$$K_{max} = \frac{1}{2}\rho\Omega^2 \int_0^R \int_0^{\frac{\pi}{2m}} w^2(r)\, n.\sin^2(m.\varphi) r.d\varphi = \frac{1}{2}\frac{\rho h}{m}\Omega^2 \int_0^R w^2(r) \int_0^{\frac{\pi}{2}} \sin^2(m\varphi)\, d(m\varphi) \tag{4}$$

And because:

$$\int_0^{\frac{\pi}{2}} \sin^2(m\varphi)\, d(m\varphi) = \left| -\frac{1}{4}\sin(2m\varphi) + \frac{m\varphi}{2} \right|_0^{\frac{\pi}{2}} = \frac{\pi}{4} \tag{5}$$

$$K_{max} = \frac{\pi}{4}\frac{\rho.b}{m}\Omega^2 \int_0^R w^2(r) r.dr = \Omega^2 \int_0^R K(r)\, dr = \Omega^2 K$$

Where: $K(r) = \frac{\pi}{4m}\rho.b.w^2(r)\, r$ (6)

and where $b = h / 2$ is one half the thickness of the disc. It seems that for the shape curve oscillation drive $w(r)$ it is $K = const.$

Furthermore, we show that, like K_{max}, can also be the maximum potential energy U_{max} (considered segment disc) to derive the following generally valid relation:

$$U_{max} = \int_0^R U(r)\,dr = U \qquad (7)$$

For a given curve shape or the selected vibration disc $w(r)$ is again $V_{max} = V = const.$

The condition of equality:

$$K_{max} = U_{max} \qquad (8)$$

then for searched natural angular frequency of oscillation disc pays:

$$\Omega = \sqrt{\frac{U}{K}}\ [\text{rad/s}] \qquad (9)$$

Because the curve shape $w(r)$ of disc oscillation is not known in advance, we need its estimate for the purposes of calculations. For various computational shaped curve $w(r)$, we get a different, corresponding them calculated values of Ω. Rayleigh showed that of all the selected (estimated) computing curves $w(r)$ is the closest one that disc oscillation curve shape, which corresponds to the lowest value of calculated natural frequency Ω.

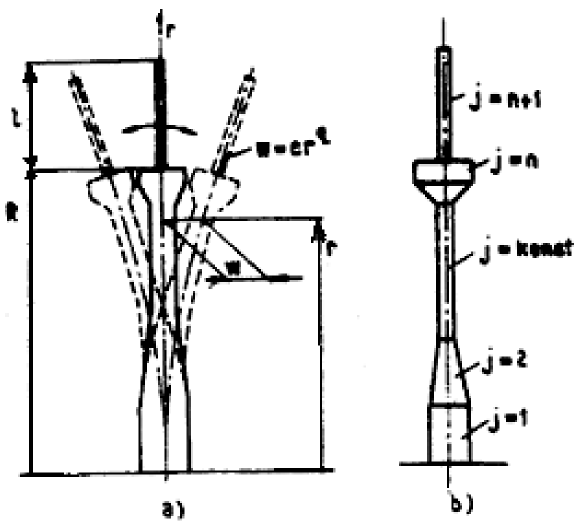

Fig. 1 Picture of the oscillation method bladed disc and the computer scheme

The curve shape $w(r)$ should be chosen to meet the boundary conditions of disc montage, including derivations. In terms of the mathematical expression of this curve it is also important to be easily derivable. For the selected suitable mathematically defined curve $w(r)$ can be derived appropriate relation by equations (6) and (7), for practical engineering calculations, whereby it is relatively easy to determine the functional dependence of $K(r)$ and $U(r)$ and after integration also associated values of constants K and U.

This computational relation will of course vary (for otherwise identical conditions), depending on the case if a disc has constant thickness or a disc has variable thickness.

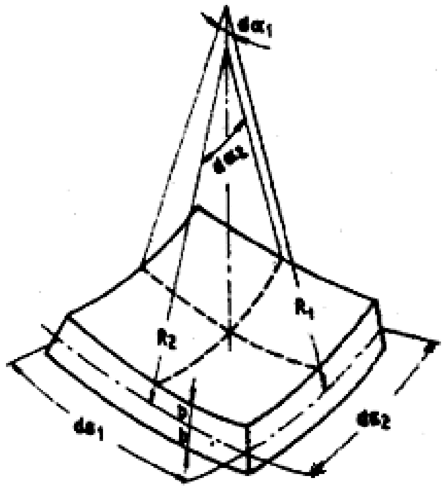

Fig. 2 Picture of the element of the disc bending

In a case of a disc with commonly changing thickness this can be replaced by finite number of rings with thicknesses constantly or linearly varied (in the direction of the radius of the tapering or expanding) and computed precisely enough (See Fig. 1b). In this case, instead of equation (9) applies:

$$\Omega = \sqrt{\frac{U}{K}}\ \text{kde:}\quad U = \sum_{j=1}^{n} U_j; \qquad K = \sum_{j=1}^{n} K_j \qquad (10)$$

Where *"n"* is the total number of disc annular elements and K_j, U_j are they relevant variables.

In case of a rotating bladed disc, instead of true equality (10) the following relationship is valid:

$$\Omega = \sqrt{\frac{U+W}{K}} \quad \text{kde:}\quad U = \sum_{j=1}^{n} U_j; \quad W = \sum_{j=1}^{n+1} W_j; \quad K = \sum_{j=1}^{n+1} K_j \qquad (11)$$

Where U is potential energy of disc without blades, consisting of *"n"* computational rings. W is the work of disc centrifugal forces, including the vane ring at the periphery (hence the index "$n+1$") and K is kinetic energy of the disc and blades – similar to the work of centrifugal forces. According to some theories is neglected the contribution of the bladed rim to the total potential energy of the disc. Kinetic energy and work of centrifugal forces of the rim are respected by so called fictitious ($n+1$) disc ring (see Fig. 1b). The outer radius of the ring is the same that the inner radius of the blades and the inner radius of the ring is equal to the outer radius of the last n^th ring of the disc, which is located close to bottom part of the blade lock. The thickness of this fictitious ring is calculated from the condition of equal masses of blades (including ridges and hinges drive) with a mass of the ring. To overcome complicate calculation, it is assumed that the disc and the blade, ie ($n+1$) have a common ring (ie, mathematically equivalent) curve $w(r)$. Of course there might be other, more or less accurate calculation assumptions, how it is possible to consider the mass distribution and stiffness of the blade on disc oscillation.

For example, it would be more accurate. to replace the blade with two calculation rings: the first one in the hinge and the second one in the leafy part. If blades are bandaged on their ends, it would be more accurate to choose even three spare blades ring. In equation (11) for the calculation of W and K would instead of ($n+1$) was ($n+2$) and ($n+3$).

From the foregoing, it is useful to derive the necessary relations for calculation of potential and kinetic energy of oscillating disc (or its parts) for commonly defined deformation curve $w(r)$, whose shape can be changed by suitably chosen parameters. Therefore we first derive a functional relation $K(r)$ and $U(r)$ in commonly applicable equations (6) and (7).

The specific procedure to calculate the natural frequency of the disc $\Omega = \Omega\ (m)$ at $m = const.$ is as follows:

1. Chosing value $q = q_{min} = const.$
2. Calculating the constant $M = M\ (m, q)$
3. Drawing U_j and gradually K_j for $j = 1,2,3, \dots n.$
4. By using equation (10) is declared U and K, and finally Ω, where $\Omega = \Omega\ (q)$
5. Repeating the calculation of $\Omega = \Omega\ (q)$ from the previous point for all other values of $q = const$. The chosen range from q_{min} to q_{max} after step $AQ = const.$
6. Illustration of relation $\Omega = \Omega\ (q)$ (see Fig. 3) and using this to determine $\Omega\ (q) = \Omega\ (q) = min\ \Omega\ (m)$, which is considered to be searched natural disc frequency for a given number of nodal diameters m.

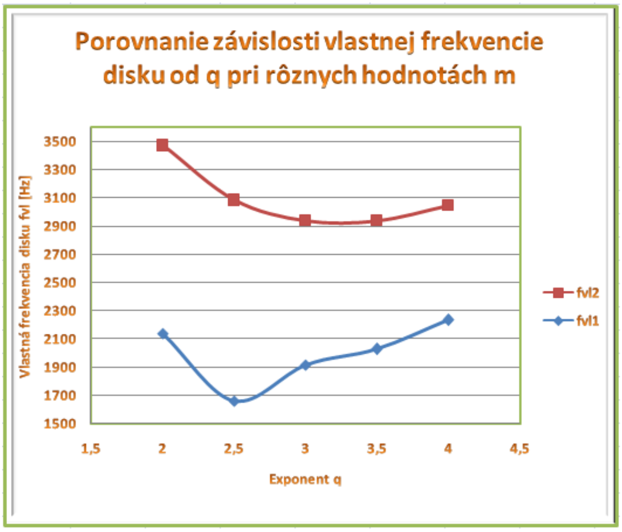

Fig. 3 Comparison of natural frequency dependence of the disk exponent q, the value of m = 4 (fvl1) and m = 5 (fvl2)

3 CONCLUSION

From the results of natural frequency of oscillation disc calculating it can be seen that for each m (number of nodal lines) is found a different value of the exponent q, which affects the curve form of the function $w\ (r)$.

After a first approximation would be appropriate to compress calculation for different q with smaller step $AQ = const$. By this would been found accurate values for finding curve $w(r)$.

In the thesis we dealed with the basic procedure for calculation of natural frequency of disc vibration that can be developed with considering the effect of centrifugal forces, influences of the shape (cross-section of the radius), disc temperature (changed E along the radius), etc.

BIBLIOGRAPHY

[1] LINHART Z. – KAMENICKÝ J.: Konstrukce Leteckych Motoru II, Pevnost a dynamika

AUTHOR(S)´ ADDRESS(ES)

Horváth Michal, Eng.
Štúrova 7, Závadka nad Hronom 976 67
horvath.polomka@gmail.com
Fábry Stanislav, Eng., PhD.
Letecká fakulta TUKE
Rampová 7, 040 21 Košice
stanislav.fabry@tuke.sk