GAS TURBINE DISC STRESS CALCULATION

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Safety running of the aircraft gas turbine engine is based on rotor components reliability. One of the key engine components is the rotating disc of the compressor or turbine. There are some calculation methods of disc stress. In the paper the method of final differences of disc dimensions for stresses calculation in radial and tangential direction was applied.

1 INTRODUCTION

Damage of aircraft gas turbine engine rotor in running, means a stoppage of the engine and subsequently an aircraft power plant failure. For the reason of safety, the probability of rotor failure must be kept to minimum and therefore at work there must be paid enhanced attention to the calculation of the gas turbine rotor disk tenseness. Factors which have a cardinal impact on the disk tenseness are being analyzed in the introductory part. The next part is analyzing the current state, also describing the methods of the disk tenseness calculations. Since there exist various methods in the area of elastic deformations, the calculation in this work is done by method of final differences.

2 GENERAL INFORMATION

The rotor disk belongs to the most stressed parts. It is stressed by centrifugal forces which rise during disk rotation acceleration, as well as from the mass of rotor blades and, in the final consequence also from thermal stresses rising during the changes in temperature in front of the turbine and behind the turbine.

In the paper I have solved calculation of stress of the turbine rotor disk, describing several methods of calculation and one of them is introduced. This method was proved at the detailed calculation of full disk by the use of the Excel program. The calculations have been performed with optimization, i.e. with repeated selection of disk parameters. The goal has been to find the way, how to solve the problem of sufficient safety coefficient and so to provide full-value materials for study of purposes at the Faculty of Aeronautics.

3 METHOD OF FINAL DIFFERENCES

This method enables to determine the disk tenseness of local shape, loaded with centrifugal forces from the weight together with external loading and thermal stress due to the thermal gradients throughout the disk radius, by means of particular calculation. At the same time it enables to include the changes of physical properties of the disk material throughout its radius in the calculation. Initial equations for calculation are equations as follows:

$$d\sigma_r = -\sigma_r \cdot \left(\frac{dy}{y} + \frac{dr}{r}\right) + \sigma_t \cdot \frac{dr}{r} - \rho \cdot \omega^2 \cdot r \cdot dr$$

$$d\sigma_t = \sigma_t \cdot \left(\frac{dE}{E} - \frac{dr}{r}\right) + \sigma_r \cdot \left(\frac{dr}{r} - \mu \cdot \frac{dy}{y} - \mu \cdot \frac{dE}{E}\right) - \\ - \mu \cdot \rho \cdot \omega^2 \cdot r^2 \cdot \frac{dr}{r} - d(E \cdot \alpha \cdot t)$$

Differentials in these equations can be replaced by final differences of disk dimensions quantities, as it is stated in Figure 1.

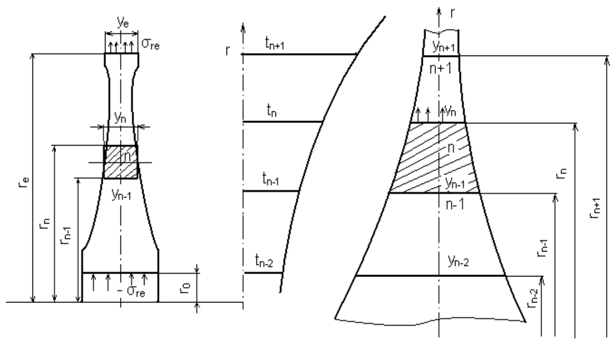

***Fig. 1** Division of the disk to the rings with individual radiuses due to the calculation by the method of final differences*

$$d\sigma_r \approx \Delta\sigma_{r_n} = \sigma_{r_n} - \sigma_{r_{n-1}} \qquad dr \approx \Delta r_n = r_n - r_{n-1}$$

$$d\sigma_t \approx \Delta\sigma_{t_n} = \sigma_{t_n} - \sigma_{t_{n-1}} \qquad dy \approx \Delta y_n = y_n - y_{n-1}$$

$$dE \approx \Delta E_n = E_n - E_{n-1}$$

$$d(\alpha \cdot t) \approx \Delta(\alpha \cdot t)_n = (\alpha \cdot t)_n - (\alpha \cdot t)_{n-1}$$

After previous equations modification by substitution of these expressions we get:

$$\sigma_{rn} = -\sigma_{rn-1} \cdot \left(3 - \frac{y_n}{y_{n-1}} - \frac{r_n}{r_{n-1}}\right) +$$
$$+\sigma_{tn-1} \cdot \left(\frac{r_n}{r_{n-1}} - 1\right) - \rho \cdot \omega^2 \cdot r_{n-1}^2 \cdot \left(\frac{r_n}{r_{n-1}} - 1\right)$$

$$\sigma_{tn} = \sigma_{tn-1} \cdot \left(1 - \frac{r_n}{r_{n-1}} + \frac{E_n}{E_{n-1}}\right) + \sigma_{rn-1} \cdot \left[\frac{r_n}{r_{n-1}} - 1 - \mu \cdot \left(\frac{y_n}{y_{n-1}} + \frac{E_n}{E_{n-1}} - 2\right)\right]$$
$$-\mu \cdot \rho \cdot \omega^2 \cdot r_{n-1}^2 \cdot \left(\frac{r_n}{r_{n-1}} - 1\right) - E_n \cdot (\alpha \cdot t)_n - E_{n-1} \cdot (\alpha \cdot t)_{n-1}$$

Due to simplification of calculation, the designation for constants is being introduced into these equations:

$$\vartheta_n = \frac{r_n}{r_{n-1}} - 1 \quad \xi_n = 3 - \frac{r_n}{r_{n-1}} - \frac{y_n}{y_{n-1}}$$
$$\varphi_n = 1 - \frac{r_n}{r_{n-1}} + \frac{E_n}{E_{n-1}} = \frac{E_n}{E_{n-1}} - \vartheta_n$$

$$\psi_n = E_n \cdot (\alpha \cdot t)_n - E_{n-1} \cdot (\alpha \cdot t)_{n-1}$$

$$\lambda_n = \frac{r_n}{r_{n-1}} - 1 - \mu \cdot \left(\frac{y_n}{y_{n-1}} + \frac{E_n}{E_{n-1}} - 2\right) =$$
$$= \vartheta_n - \mu \cdot \left(\frac{y_n}{y_{n-1}} + \frac{E_n}{E_{n-1}} - 2\right)$$

$$C_n = \rho \cdot \omega^2 \cdot r_{n-1}^2$$

Consequently equations are simplified to the shape:

$$\sigma_{rn} = \sigma_{tn-1} \cdot \vartheta_n + \sigma_{rn-1} \cdot \xi_n - C_n \cdot \vartheta_n$$

$$\sigma_{tn} = \sigma_{tn-1} \cdot \varphi_n + \sigma_{rn-1} \cdot \lambda_n - \mu \cdot C_n \cdot \vartheta_n - \psi_n$$

For the first part of the full disk ($\boldsymbol{\sigma_{rs} = \sigma_{ts} = \sigma_0}$), equations can be stated in the form:

$$\sigma_{r1} = \sigma_0 \cdot \vartheta_1 + \sigma_0 \cdot \xi_1 - C_1 \cdot \vartheta_1 = A_1 \cdot \sigma_0 + B_1$$

$$\sigma_{t1} = \sigma_0 \cdot \varphi_1 + \sigma_0 \cdot \lambda_1 - \mu \cdot C_1 \cdot \vartheta_1 - \psi_1 = M_1 \cdot \sigma_0 + N_1$$

Where:

$$A_1 = \vartheta_1 + \xi_1 \quad M_1 = \varphi_1 + \lambda_1$$

$$B_1 = -C_1 \cdot \vartheta_1 \quad N_1 = -\mu \cdot C_1 - \psi_1$$

By analogy, for the second part of the disk there it is valid:

$$\sigma_{r2} = A_2 \cdot \sigma_0 + B_2 \quad \sigma_{t2} = M_2 \cdot \sigma_0 + N_2$$

For n[th] part, it is generally valid:

$$\sigma_{rn} = A_n \cdot \sigma_0 + B_n \quad \sigma_{tn} = M_n \cdot \sigma_0 + N_n$$

Where:

$$A_n = A_{n-1} \cdot \xi_n + M_{n-1} \cdot \vartheta_n \quad B_n = B_{n-1} \cdot \xi_n + \left(N_{n-1} - C_n\right) \cdot \vartheta_n$$

$$M_n = M_{n-1} \cdot \varphi_n + A_{n-1} \cdot \vartheta_n$$

$$N_n = N_{n-1} \cdot \varphi_n + B_{n-1} \cdot \lambda_n - \mu \cdot C_n \cdot \vartheta_n - \varphi_n$$

Where $\mathbf{A_n}$ and $\mathbf{M_n}$ are the part constants which are the function of the disk shape resulting from physical properties of its material and they are dimensionless.

Coefficients $\mathbf{B_n}$ and $\mathbf{N_n}$ are the function of disk load with centrifugal forces and thermal gradients, their parameters are determined in MPa units.

As to the full disk, the boundary conditions in the middle of the disk for r = 0 is $\boldsymbol{\sigma_{r0} = \sigma_{t0} = \sigma_0}$ and the application from the equation is as follows:

$$\sigma_{r0} = A_0 \cdot \sigma_0 + B_0 \quad A_0 = 1 \quad B_0 = 0$$

$$\sigma_{t0} = M_0 \cdot \sigma_0 + N_0 \quad M_0 = 1 \quad N_0 = 0$$

Because of unknown A_0, B_0, M_0, N_0, the values A_1, B_1, M_1, N_1 are calculated on the radius $\mathbf{r_1}$ and on other radiuses up to the radius $\mathbf{r_e}$, where A_e, B_e, M_e, N_e come from equations.

By means of substitution of particular constants there is calculated the main stress in individual parts as a function of unknown value of the stress $\boldsymbol{\sigma_0}$ in the middle of the turbine rotor disk. On radius $\mathbf{r_e}$, there is known the value of stress $\boldsymbol{\sigma_{re}}$ from the centrifugal force of blades and one part of the disk rim, and it is valid, according to above equations that $\boldsymbol{\sigma_{re} = A_e.\sigma_0 + B_e}$, where there is possible to determine the unknown value of the stress:

$$\sigma_0 = \frac{\sigma_{re} - B_e}{A_e}$$

By means of substitution of the value $\boldsymbol{\sigma_0}$ into the expression for stress of individual parts, there is possible to calculate specific values of the stress throughout the whole disk radius. For the disk with central hole, the initial calculating radius is $\mathbf{r_0}$, i.e. the radius of the hole. For $\mathbf{r = r_0}$ will be $\boldsymbol{\sigma_{r0} = 0}$ and for the disk non-fixed on a shaft (practically it is almost always) $\boldsymbol{\sigma_{t0} = \sigma_0}$. The part constants on the radius $\mathbf{r_0}$ will be given from equation 67 as follows:

$$\sigma_{r0} = 0 = A_0 \cdot \sigma_0 + B_0 \quad A_0 = 0 \quad B_0 = 0$$

$$\sigma_{t0} = M_0 \cdot \sigma_0 + N_0 \quad M_0 = 1 \quad N_0 = 0$$

Further procedure of calculation is consistent with the calculation of the full disk. The unknown stress $\boldsymbol{\sigma_0}$ is again calculated from boundary conditions on outer disk radius $\mathbf{r_e}$, or $\boldsymbol{\sigma_{re} = A_e.\sigma_0 + B_e}$ and here is valid:

$$\sigma_0 = \sigma_{t0} = \frac{\sigma_{re} - B_e}{A_e}$$

For the disk fixed on a shaft is valid merely the greatness of constant $\boldsymbol{B_0 = -\sigma_{r0}}$. For sufficient accuracy of calculation it is suitable to choose the number of calculating sections more than 7, and their density depends on a nature of the change of gas turbine disk thickness. For the disk with central hole there is suitable, close to the hole, to choose thicker division, with regard to the rapid change in stress $\boldsymbol{\sigma_{t0}}$, $\boldsymbol{\sigma_{t1}}$. It is suitable to perform the calculation by way of use the table calculator.

4 CONCLUSION

For purpose of adequate accuracy of calculation it is suitable to choose the number of calculating sections higher than 7. Their density is dependent upon the nature of the disk thickness change. It has been performed the calculation by the use of the table. Based on consistency during gradual recording of calculation algorithm in the table calculator there have been removed the inaccuracies in considerations that have been found in used literature. It has to be told that the goal of work has not been a design proposal of a gas turbine rotor disc, but the partial strength inspection for fictitious disk and supporting calculations that are being performed during planning and designing of a rotor disc. In the last part of the diploma thesis is briefly described the course of calculation. There is presented a simple chart of the disk calculated as well as resulting coefficients of safety. The coefficient of safety has been calculated by comparison of the calculated reduced stress throughout the disk radius with allowable long-time strength.

Since the rotor disk belongs to the most stressed parts of the engine, the entire calculation has been directed towards achieving the disk as light as possible with acceptable safety, and it has been aimed to fulfill the conditions so that the safety coefficient cannot fall below the 1,5 limit.

The attached procedure of calculation enables to repeatedly perform individual calculations of the disk stress for the needs of engineering study.

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