VERIFICATION OF MATHEMATIC MODELS OF A SMALL TURBOJET ENGINE MPM - 20

Lucia Bujňáková – Ladislav Főző

Mathematical modeling is one of the most common ways of acquiring information and the properties of the object under examination. When changing the parameters entering the model can be obtained depending on the course and characteristics of the individual parts and the whole of subjects, which is the aviation turbojet engine MPM20. For this engine were created experimentally, graphical-analytical models and analytical a first level, whose analysis and verification is the goal of this post. Mathematical models of the engine are required to use its management, diagnostics and failure. Thanks analytical model and entering static characteristics, it is possible to calculate the parameters that are not measurable or difficult to measure.

K e y w o r d s: mathematical model analysis, verification, aircraft turbojet engine MPM 20

1 INTRODUCTION

Thanks to the mathematical model, we are able to manage aircraft engines more reliable, more accurate and more economical, and for the digital management is possible in the beginning to identify possible failures of sensors, or the entire building. These models must constantly improved and refined. Also because this work is devoted to analysis, verification and possible adjustment of the motor mathematical model MPM20, namely mathematical experimental model, graphical-analytical and analytical model of the first level. In the present work is described in detail analytical model of the first level, which is based on the approximation of the characteristics of the individual parts of the engine and the flow of working fluid, which are described by algebraic and differential equations applied to the motor MPM20. In the last section summarizes the verification of the above mathematical models based on the advantages and disadvantages of each model.

2 BASIC INFORMATION

Mathematical modeling is the most common method of obtaining information and the properties of the object under examination. In mathematical modeling when changing parameters can be achieved by the availability of different variants of subjects in a very short time. Mathematical model of equilibrium and disequilibrium running (theoretical elaboration in publications [1]) air turbojet engine (ATJE) is based on the embodiment of universal physical laws that characterize the properties and activities of individual nodes and parts of the engine, the working fluid properties and thermodynamic and aerodynamic going with consideration changes in the properties of the working fluid. In doing so, it is necessary to take into account a range of working conditions, aircraft engines, which is characterized by working range of heights and speeds of flight, i.e. changing the thermodynamic properties of the working fluid.

3 CHARACTERISTIC OF THE ENGINE MPM20

Air turbojet engine MPM20 is small, single-jet, single-shaft engine, created a design modification aviation, turbine starter TS-20. Starter TS-20 was originally used to spin the rotor ATJE on earth during its startup or cold twisted, conservation or depreservation. It was adapted to the needs of experimental measurements, currently is located in the premises of Faculty of Aeronautics Technical University of Kosice. This engine due to its small size is suitable for laboratory measurements, because its characteristics are very similar in size to the large engine uncomplicated structure compared to the motors of classical conception and economic access. Engine MPM20 is a small jet engine with a single-stage, centrifugal compressor with single impeller, an associated combustion chamber, single stage axial, uncooled gas turbine reaction type and output system with fixed output nozzle. The scheme of the air engine MPM20 can be seen in Fig. 1. This engine contains a separate fuel-regulatory, oil and electric trigger system [1].

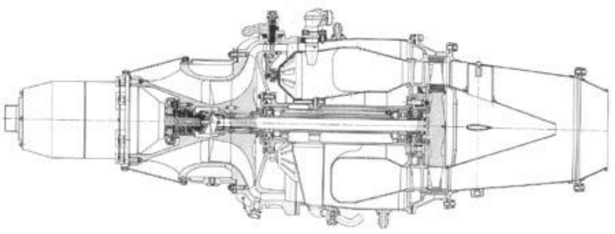

Fig. 1 Small jet engine MPM- 20

4 ANALYSIS OF MATHEMATICAL MODELS ATJE MPM20

Modeling, identification and control of complex systems progressive methods have previously been in the spotlight. In connection with this issue have been developed and applied various modern methods and approaches of management objects based on artificial intelligence. The next section will analyze created mathematical models applied to the engine MPM20 and experimental, analytical and graphical-analytical model of the first level.

4.1 Experimental model ATJE MPM20

The experimental model is based on experimental analysis of conduct examined system (object). Experimental identification based on measured values of the system, i.e. of evaluating information on input and output parameters of the experiment and their mutual relations. The structure of the model must be chosen in advance in experimental identification. The parameters of such a model are analytical variable and usually have no connection with the physical system variables. Internal processes in models may not be known, but experimental model can be created only for the existing real object.

4.2 Graphical-analytical model ATJE MPM20

For designing the graphical-analytical model was necessary to create a system for dynamic changes the operating mode of a small jet engine MPM20, which is controlled by the computer and is operational in real time, i.e. the engine running. For non-measurable parameters was proposed estimator (estimate), based on an experimental model engine MPM20 obtained by method of progressive iteration. From the measured data and thermodynamic calculations of the parameters of the thermal cycle was developed analytical model engine MPM20 by creating and evaluation of mathematical models of the individual parts of the engine (inlet system, compressor, combustor, turbine, and output system), which was subsequently determined curve joint work all parts of the engine and creating a mathematical model of steady (steady) and transient (unbalanced) motor running MPM20 with fixed and variable geometry.

Great emphasis was placed on mathematical models and characteristics of the various engine parts MPM20 from which further unfolded the accuracy of the model equilibrium and disequilibrium mode of engine. Mathematical model of steady and unsteady modes engine is based on the embodiment of universal physical laws that characterize the properties and activities of individual nodes and parts of the engine, the working fluid properties. Were taken into account the thermodynamic and aerodynamic going with consideration changes in the properties of the working fluid.

In determining the *curves of joint work* of all parts of the engine for any mode of the engine has to pay continuity equation. This equation determines the relationship between the mass flow rate of air compressor, combustor, turbine, and output system. First, the desire to meet the conditions of continuity equation for all parts of the engine, provided that the known characteristics of all parts of the engine.

4.3 Analytical model of the first level ATJE MPM20

Author of monograph [1] discusses in general on the establishment of a mathematical model of a jet engine (JE), which can be applied to the design procedure of the mathematical model ATJE MPM20. Mathematical model of the first level is described to the extent possible in similarity, and dimensionless parameters relative to the use of a mathematical model for optimization calculations for design and research and development tasks.

In general characteristics of the main parts of the PM divided into geometric, which reflect the main dimensions of the JE and gas-dynamic characteristics. The following description will be understood by just gas-dynamic characteristics that express the dependence of the main gas-dynamic and thermodynamic parameters of JE parts in the mode of work. Characteristics parts of ATJE, the engine MPM20 talking about the input system, a compressor, a combustion chamber, a gas turbine and output system, it is possible to obtain the mathematical modeling on the basis of tests of these parts, or by calculation from the known structural and geometrical parameters of parts, or by calculation from the general characteristics the individual parts.

For steady operation ATJE apply four basic terms and conditions that I applied for a single-engine small MPM20 and chose four specific, namely:

1. The condition of constancy mode - power turbine and power compressor input is equal to:
$$P_T.\eta_{mech} - P_K = 0 \tag{1}$$
2. Mass flow balance condition:
$$Q_{T4,char}(\pi_{Tc}, \lambda_u) - Q_{T4} = 0 \tag{2}$$
3. The condition law of engine control - section of a nozzle is unchanging:
$$A_{TR} - A_{TR,v} = 0 \tag{3}$$
4. The condition of fuel supply is required by law:
$$q_{pal} - q_{pal,vyp} = 0 \tag{4}$$

Equation (1), (2), (3) and (4) consists of four conditions steady mode engine MPM20. These are then converted into dimensionless form:

$$f_1 = 1 - \frac{P_K}{P_T} = 0 \tag{5}$$

$$f_2 = 1 - \frac{Q_T}{Q_{T,char}(\pi_{Tc}, \lambda_u)} = 0 \tag{6}$$

$$f_3 = 1 - \frac{A_{Tr,vyp}}{A_{Tr}} = 0 \tag{7}$$

$$f_4 = 1 - \frac{q_{pal,vyp}}{q_{pal}} = 0 \tag{8}$$

where:

P_K - compressor power

P_T - power of the gas turbine

Q_T - mass flow of the inlet gas to the gas turbine, that is, the mass flow at the outlet of the combustion chamber

$Q_{T,char}(\pi_{Tc},\lambda_u)$ - mass flow of gas from the gas turbine is determined characteristics of the gas

$A_{Tr,vyp}$ - entered value of the flow area of the outlet nozzle

A_{Tr} - a flow area of the outlet nozzle determined from the condition of continuity and pressure gradient at the nozzle

$q_{p,vyp}$ - entered relative supply of fuel to the combustion chamber

q_p - the proportion of fuel supply to the combustion chamber as determined from the calculation of the thermodynamic

Modified equations are used to find the steady running of the engine, so finding such a combination of independent options (variables) parameters, for that would ensure the validity of the above-mentioned conditions.

For solving systems of nonlinear algebraic and differential equations consisting of the previous conditions, it is necessary to design the engine MPM20 and four optional parameters. I chose the following optional parameters:

- On the characteristics of the compressor: *mass flow* and *speed* - two independently selectable parameters
- *Supply of fuel* into the combustion chamber
- The position of the operating point on the characteristic of the turbine - the choice of *pressure gradient*

When looking for steady operation engine is basically to find such a combination of four independently selectable parameters, which ensure compliance equations (5), (6), (7) and (8). This equations and parameters I rewrote the successive matrices:

$$x = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Vmatrix} \tag{9}$$

where:

$x_1 = \overline{n_k}$, $x_2 = q_k$, $x_3 = q_p$, $x_4 = \pi_{Tc}$

$$F = \begin{Vmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Vmatrix} \tag{10}$$

where:

f_1 – f_4 are equations of equilibrium mode in dimensionless shape

The first step is to estimate as accurately independently selectable parameters. With these estimated parameters takes place thermodynamic equilibrium calculation engine running, while the gain variation equations $f_1 – f_4$, which I have identified: Δf_j^1, where j =1–4 for four equations f. Next, edit parameter x_1: $x_1 = 1{,}001 * x_1$. Using this value x_1 and with original value x_2 – x_3 again recalculates the thermodynamic calculation of equilibrium mode engine, while I get other error for the equation f_1 – f_4, which I identified subsequently: Δf_j^2, where j =1–4. Then I calculated the numerical derivatives, i.e. effects of the independent optional parameter equation $f_1 – f_4$ using the formula:

$$\frac{\Delta f_j^2 - \Delta f_j^1}{0{,}001 * x_1} \tag{11}$$

Once calculated deviations using the formula (11) for four equations f_1 – $f_{4,}$ is value x_1 returns to its original value and change the value x_2: $x_2 = 1{,}001 * x_2$. With this altered value and other original value prairie again thermodynamic calculation. Repeat this procedure for the x_3 a x_4. Then I can add Jaccobiho matrix:

$$J = \begin{Vmatrix} \frac{\Delta f_1^2 - \Delta f_1^1}{0{,}001.x_1} & \frac{\Delta f_1^2 - \Delta f_1^1}{0{,}001.x_2} & \frac{\Delta f_1^2 - \Delta f_1^1}{0{,}001.x_3} & \frac{\Delta f_1^2 - \Delta f_1^1}{0{,}001.x_{41}} \\ \frac{\Delta f_2^2 - \Delta f_2^1}{0{,}001.x_1} & \frac{\Delta f_2^2 - \Delta f_2^1}{0{,}001.x_2} & \frac{\Delta f_2^2 - \Delta f_2^1}{0{,}001.x_3} & \frac{\Delta f_2^2 - \Delta f_2^1}{0{,}001.x_4} \\ \frac{\Delta f_3^2 - \Delta f_3^1}{0{,}001.x_1} & \frac{\Delta f_3^2 - \Delta f_3^1}{0{,}001.x_2} & \frac{\Delta f_3^2 - \Delta f_3^1}{0{,}001.x_3} & \frac{\Delta f_3^2 - \Delta f_3^1}{0{,}001.x_4} \\ \frac{\Delta f_4^2 - \Delta f_4^1}{0{,}001.x_1} & \frac{\Delta f_4^2 - \Delta f_4^1}{0{,}001.x_2} & \frac{\Delta f_4^2 - \Delta f_4^1}{0{,}001.x_3} & \frac{\Delta f_4^2 - \Delta f_4^1}{0{,}001.x_4} \end{Vmatrix} \tag{12}$$

Matrix new choice independently variable parameters provided me with:

$${}^2x = {}^1x - \left[J\left({}^1x\right)\right]^{-1}.F\left({}^1x\right) \tag{13}$$

where:

${}^2x$ - new matrix independently selectable parameters

${}^1x$ - original matrix independently selectable parameters

$\left[J\left({}^1x\right)\right]^{-1}$ - inverse matrix of Jaccobiho matrix (12)

$F\left({}^1x\right)$ - matrix of deviations $f_1 – f_4$, for original value $x_1 – x_4$

This iterative cycle for the calculation parameters and equilibrium mode engine and deviations equations repeat until the condition – criterion termination calculating, which is defined by the following equation (14) and for my selected value for $\varepsilon = 10^{-6}$.

$$\varepsilon = \sum_{j=1}^{10} \sqrt{\Delta f_j^2} \tag{14}$$

On the Fig. 2 is shows a flowchart of an algorithm to calculate the above independently of optional parameters.

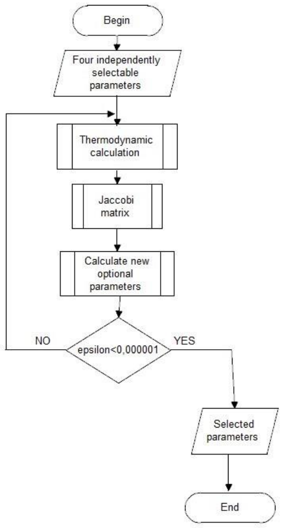

Fig. 3 Flowchart for calculating matrix Jaccobiho

Mathematical model that describes the flow of working fluid is formed by the JE description of the thermodynamic properties of the working fluid, any changes to the working fluid, describing the flow of working fluid gas-dynamic functions describing the flow of working fluid different parts JE and describes the properties of the control system.

5 VERIFICATION AND EVALUATION OF MATHEMATIC MODELS

In experimental models, compared to the speed of analysis is an advantage. These models is needed less time, since we do not need witch calculates analytical parameters. This makes another advantage and that is that we do not need to know the internal processes and links between variables. Disadvantage of these models is needed but the real object on which experiments are carried out. Measured data and parameters are based on experimental models. Another problem in the experimental analysis may be that not all the necessary parameters can be measured or of the experiment to obtain. When repeating these experiments may result in damage to the engine, or parts thereof, which causes inaccuracy, or impossibility of further experiments and researches.

In the second, graphical-analytical model is a combination of input parameters. One part of the parameters is calculated from thermodynamic calculation and the rest of the experimental model and estimation. The advantage of this model is to use graphic program ANFIS, which replaces the time-consuming calculations to determine the characteristics and approximations of individual nodes engine that underlie the dynamic model of the characteristics of collaborative work and steady engine running at significantly less time. The analytical part we use the calculate parameters cannot be measured.

The third group of mathematical models, which I have addressed in this work is an analytical model of the first level. This model is further detailed. It uses a modified and applied differential and algebraic equations. The biggest advantage of these models is the ability to calculate all the necessary parameters and those measured in the engine not using thermodynamic calculations and approximations according to the characteristics of appropriate mathematical methods. The disadvantage is the slowness of these calculations and the need for continuous improvement and refinement of mathematical models. The disadvantage is the large range of working conditions, aircraft engines, which are characterized by a wide range of parameters of height and airspeed, i.e. changes in the thermodynamic properties of the working fluid, which complicates the already time-consuming calculation.

Any mathematical model, whether experimental, graphical-analytical or analytical model of the first level, have their shortcomings. Although their use has a huge contribution to the diagnosis as well as in the management of aircraft engines, the engine MPM20 too.

5 CONCLUSION

Theoretical knowledge on the procedure for obtaining the mathematical model of the first level I applied for a small current ATJE MPM20. Using a suitable approximation methods - classical polynomial approximation (least squares method), it is possible to approximate the characteristics of individual nodes (parts) engine. Formula underlying mathematical model describing the flow of working fluid motor in terms of thermodynamic calculation and gas-dynamic functions. Description of flow of the working fluid is created separately for each part of the engine (input device, compressor, combustor, gas turbine output device). The mathematical model for engine MPM20 are four independently selectable parameters. I chose the relative speed and the compressor mass flow, fuel delivery to the combustion chamber and the pressure gradient of the gas turbine. Mathematical model for the functionality you need to select a suitable combination of these parameters for which must be met four conditions of steady engine running MPM20. Calculation of optional parameters I dealt with as a system of nonlinear algebraic and differential equations using the modified Newton method. Optional parameters independently affect the deviation equation expresses Jaccobiho inverse matrix effects, thus replacing the analytical derivation of numerical differentiation, which I applied for the calculation too. When compared to other engine models, mathematical model of the first level is particularly advantageous when calculating immeasurable, or hardly measurable parameters. This model after adjustment is necessary to converge to MATLAB / Simulink so that it can be used in the diagnosis and to enable the model to run parallel with the real running engine.

BIBLIOGRAPHY

[1] POVAŽAN, Jozef: Konštrukcia matematických modelov leteckých turbokompresorových motorov. Košice: 1999.

[2] HOCKO, Marián: Malý prúdový motor „MPM 20“. Košice: VLA M.R.Š., 2003.

[3] BELLOM, Nicola – De ANGELIS, Elena – DELITALA, Marcello: Lecture notes on Mathematical Modelling in applied sciences. Torino, Italy: 2007.

[4] FÁBRY, Ján: Matematické modelovaní. Praha: 2011

[5] FŐZŐ, Ladislav: Využitie matematického modelu rovnovážneho a nerovnovážneho chodu motora MPM20 pri návrhu algoritmu riadenia v každom čase. Doktorandská dizertačná práca, FEI TU Košice, 2008

[6] ŠUTÁK, Martin: Modernizácia systému automatického riadenia generátora vzduchu Saphir-5. Doktorandská práca, LF TU Košice, 2010