LAWS OF RELIABILITY

Jozef Himič – Ján Kabát

The article is a treatise on the application of several kinds of laws when dealing with reliability not only in theory but practice, as well.
K e y w o r d s.: exponential law, normal/standard law.

1 INTRODUCTION

Reliability of products is currently understood as an integral part of a whole sum of its characteristics affecting its ability to meet the stated and assumed need s of the user. This capability is summed up as quality. Apart from reliability, quality also involves a number of further partial characteristics of objects.

The notion of reliability is used only for general description and cannot be quantified and explained by of numerical indicators. However, the individual partial indicator – readiness, faultless operation and sustainability can be quantified and evaluated by means of concrete indicators.

In order for us to determine the functions of reliability applying the methods of statistics, a great many of test must be performed. Apart from it, these tests must be performed within a certain time interval (period), which causes information obtained on the reliability of equipment or elements to be valid only in the period of time, whereas there is no information available regarding reliability behind the time limitations.

2 BASIC MONITORING OF RELIABILITY INDICATORS

Indicators of reliability are understood as tools that enable description of stochastic phenomena and processes, which characterize the reliability of objects.

Basic knowledge from the theory of probability coupled with the issue of indicators of reliability say that with each variable (defect) is related to certain rule, which helps determine the probability of expecting the occurrence of a phenomena (occurrence of defect). This rule is called the law of distribution of the probability of random variable [1, 2, 4].

Indicator of reliability can be:

- Function of the law of distribution of the monitored failure (distributive function, density of probability, intensity of failure...),
- Numerical characteristics of the distribution of probability of the monitored defect (mean value, dispersion, standard deviation...).

In practice a whole range of indicators is used, the selection of which is made so as to obtain the best possible characterisation of the object, way of it's us and operational conditions.

3 EXPONENTIAL LAW

This law is suitable for the description of faultlessness at the occurrence of random failures, which more frequently occur under heavy duty operations. It is also suitable for complex systems, which have different $\lambda(t)$ characteristics or have passed trial run [2, 3].

a) Probability of faultless operation;

$$R(t) = e^{-\lambda t} \tag{1}$$

λ – parameter of failure intensity
t - time

b) Probability of failure;

$$Q(t) = 1 - e^{-\lambda t} \tag{2}$$

c) Density of failure distribution;

$$f(t) = \lambda\, e^{-\lambda t} \tag{3}$$

d) Intensity of failure;

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1-Q(t)} = \frac{\lambda . e^{-\lambda t}}{e^{-\lambda t}} = \frac{1}{T_S} \tag{4}$$

It is dependent on time, which is typical feature of the exponential law.

e) Mean time of faultless operation

$$T_S = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda} \tag{5}$$

Then equation (1) can take the form

$$R(t) = e^{-\frac{t}{T_S}} \tag{6}$$

f) Standard deviation δ

expressed in the form of an initial moment of second grade α_2 takes the form

$$\delta = \sqrt{\alpha_2 - T^2} = \sqrt{\frac{2}{\lambda^2} - \frac{1}{\lambda^2}} = \frac{1}{\lambda} = T_S . \tag{7}$$

Simultaneously, it is equal to the mean time between failures, which is the second typical characteristics of the exponential law.

Exponential law is of extreme importance in the theory of reliability. In most cases it can be used where relatively complex mathematical problems are solved on condition that the distribution function will demonstrate exponential behaviour. It is important to realize when it is justified to apply the exponential law and know in which situations the exponential distribution will appear.

The known cases of applying the distribution:

1. Hypothesis on exponential distribution of the service life of the element can be described statistically. For many of the elements types in the wide area of statistical material, there is the following shape of the failure intensity curve, see Fig. l.

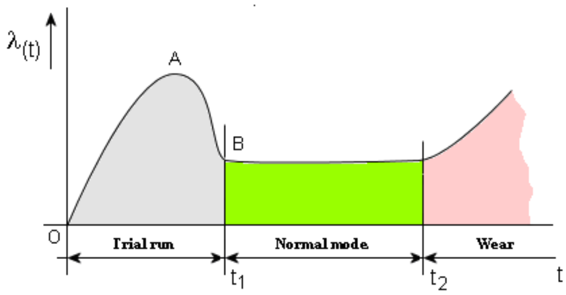

Fig. 1 Shape of the failure intensity curve

By nature, intensity of failure behaviour, the entire time axis can be divided into three parts:

a) In part $(0, t_1)$, the intensity of failures is growing increasingly. This part is termed as the period of trial run. This growth is explained that, in any arbitrary group of elements, at testing, right in the beginning the failures are indicated at elements that are carriers of certain defects.
b) Within the time interval of (t_1, t_2), intensity of failures remains roughly at the same – constant level. This time interval is called as the period of normal operational activity.
c) The last time interval (t_2, ∞) is also termed as the period of ageing. Within this time period, in the elements, physic – chemical changes take place, thereby the elements are ageing, their rate of wear is relatively high and with reliability the intensity of failures is increasing monotonously.

At many types of elements, the period of ageing start following a substantial long period of time, and the period of activities belongs to the normal period, where the intensity of failures is relatively at a constant level. In such case, when calculating the characteristics of reliability, one can apply the exponential law.

At statistical verification of the exponential distribution two conditions are to be respected:

- The number of experiments must be satisfactorily big, so as to achieve number of failures recorded by order of hundreds.
- If experiments are performed within a certain time interval, then the confirmation of the exponential distribution can also be performed only wit based on that time interval. On cannot assert that the exponential distribution is also beyond this interval, should the number of experiments performed be very small.

2. In case when the only cause of the element failure is e-g- disconnection of contacts due to vibrations. Changes in the forces of vibrations in time mostly resemble a rapidly oscillating stationary process. If the amplitude of the given process surpasses a certain critical level, contacts get disconnected. Even in such case, one can apply the exponential law. In certain cases, it is also possible to verify also physically the correctness of the equation, from which the exponential distribution is apparent.

3. If a complex system has no back-up elements, then the failure of the element turns into the failure of the entire system. In case of failure, the non-functioning element is replaced by a functional one. Then the flow of failures of the given system will be the sum of failure flows of the individual elements. If the equipment is made up of a large number of renewable elements, and if such equipment has been working for a sufficiently long period of time, then the time period from the moment till the equipment (system) failure with good approximation will have an exponential distribution.

When applying exponential distribution to solve problems within the theory of reliability, the rules underlying of the approach are to be necessarily verified in each case.

4 NORMAL LAW

Among various laws of distribution of analogous random variables, a special position is held by the law of normal distribution. In practice, we often come across with normal distribution, for example at defects of products, measurements of spare parts, dispersion of mechanical characteristics of materials and the like [1, 2].

Density of distribution at normal law can be expressed by the following equation:

$$f_{(x)} = \frac{1}{\delta.\sqrt{2.\pi}}.\left\{\exp-\frac{\left(x-\bar{x}\right)^2}{2.\sigma^2}\right\}. \qquad (8)$$

$\bar{x}$ - mean value (mathematical expectation),

δ - mean quadratic deviation of the analogous random variable x.

Figure 2 illustrates the density of the normal distribution in accordance with the mathematical expression (8).

Let us assume that at normal distribution, the random variable x can take the values: $-\infty\langle x\langle\infty$, whereas the probability of larger deviations is very low.

Normal distribution is of dual-parameter -type, (giving two parameters: $\bar{x}$ and δ) by its density it determines the distribution. Parameter $\bar{x}$ is truly a mean value (mathematical expectation) of the random variable x. Then, mathematical expectation takes the following form:

$$\bar{x} = \langle x \rangle = \int_{-\infty}^{\infty} x.f(x)dx = \frac{1}{\delta.\sqrt{2\pi}}.\int_{-\infty}^{\infty} x.e^{-\frac{(x-\bar{x})^2}{2.\delta^2}}.dx \cdot (9)$$

Introducing normal/standard deviation:

$$u = \frac{(x-\bar{x})}{\delta}. \qquad (10)$$

This deviation plays an important role in the theory of probability and in mathematical statistics. Physical importance of the standard deviation is as follows: value **u** is a random variable, to which the distribution by the normal law wit a zero mean value is applied.

$\bar{u} = 0$, an quadratic deviation.

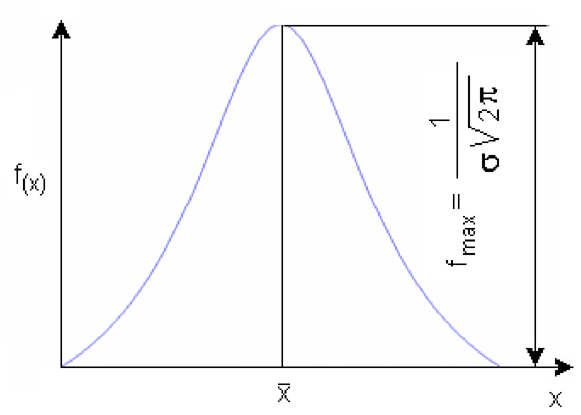

Fig. 2 Density of normal distribution

In the theory of reliability, application of the cut-down normal law is known, the mathematical form of which applied to probability of faultless operation is as follows:

$$R\{\zeta \geq t\} = \frac{c}{\delta.\sqrt{2.\pi}}.\int_{-\infty}^{\infty} \exp\left[-\frac{(x-t)^2}{2.\delta^2}\right]dx \cdot \qquad (11)$$

Where coefficient **c** is determined from the condition: $R_{(0)} = 1$. In the real time of service life distribution, the mean quadratic deviation is much smaller than the mean service life, therefore we can chose c = 1 and then M. ζ = T and D. ζ = δ^2.

Intensity of failures for normal law takes the form, which is graphically presented in Fig. 1.

Curve $\lambda_{(t)}$ miss growing monotonously and is approaching the asymptote:

$$y = \frac{(t-T)}{\delta}. \qquad (12)$$

As it has already been stated, a known distribution of all failures is the following:

- **Abrupt failures**, which are of random nature and are subject to law of exponential distribution,
- **Gradual failures**, which arise as a result of ageing and gradual wear of the elements and are subject to the law of normal distribution.

Generally, the normal law can be used (with considerable precision), if the density of failures f(x) has a simultaneous symmetrical shape and δ 〈〈 T.

In that case, a model can be constructed in which the normal law is being generated in a natural way.

5 DETERMINING THE VALUES OF THE RELIABILITY CHARACTERISITICS AT GRADUAL FAILURES

Use of technical equipment in practice results in gradual occurrence of failures, caused by wear and ageing. To such equipment, the law of normal distribution is mostly applied. [1, 5]. In view of the fact that the time value variable of the faultless operation is a positive value, the frequency of failures can be determined in the following way:

$$a(t) = C.\exp\left[-\frac{(t-T_S)^2}{(\delta^2)}\right], \qquad (13)$$

C – constant of the cut-off normal distribution, which can be determined from the condition:

$$\int_0^{\infty} a(t)dt' = 1, \qquad (14)$$

T – mean value of the faultless operation,
δ^2 – dispersion.

If the mean quadratic deviation is of no great value compared to T, then the approximate computation can make use of the non-cut-off normal distribution at:

$$C = \frac{1}{(\delta.\sqrt{2.\pi})}. \qquad (15)$$

In that case, for the time period of t, the basic mathematical expression of determining the probability of element failure will be equal to:

$$Q_{(t)} = \frac{1}{\delta.\sqrt{2.\pi}} \int_{-\infty}^{t} e^{-\frac{(t-T_S)}{(2.\delta^2)}} dt \,. \qquad (16)$$

Mathematical expectation of the time period of faultless operation is approximately equal to the mean time of faultless operation of the element. This is determined experimentally, on the basis of results of the experiments for the group of elements of the same type.

Probability of the faultless operation of a group of elements, at which the monitored behaviour of gradual works will be:

$$R_{(t)} = \prod_{k=1}^{n} R_{k(t)} \,, \qquad (17)$$

n – number f subgroups with the same type of elements.

5 CONCLUSIONS

In this topic the effort was made to point out the various ways of obtaining parameters for statistical mathematical models.

Also mentioned were two laws, of which the exponential law which in the theory of reliability is of extraordinary importance.

In view of the fact that aviation equipment is ageing in the operation, it is of more advantage to make use of the law of normal distribution. Based on the mathematical results, and on the comparisons made, we are able to diagnose the momentary reliability of the aviation equipment.

BIBLIOGRAPHY

[1] HOŠKO, J. - Základy spoľahlivosti, VVLŠ Košice, 1989, 180 s. ,
[2] CALABRO, S.R.: Základy spolehlivosti a jejich využití v praxi, Knihtisk, Praha,1963,309 s. (Reliability principles and practices),
[3] TŔÍSKA, V. - Provoz a údržba letadel, VAAZ Brno 1977, 93 s. ,
[4] POLSTEROVÁ, H. - Spolehlivost v elektrotechnice, VUT Brno, 2003, 86 s.,
[5] SCHINDLER, R.: Prevádzková spoľahlivosť lietadlových konštrukcií, VŠDS Žilina, 1983

AUTHOR´S ADDRESSES

Himič, Jozef, Ing.
Technical university of Košice
Faculty of aeronautics
e-mail: jozef.himic@tuke.sk

Kabát, Ján, Ing., PhD.
Technical university of Košice
Faculty of aeronautics
e-mail: jan.kabat@tuke.sk