CYCLICAL SURFACES CREATED FROM SINUSOID ON SURFACE OF REVOLUTION

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The aim of this paper is a creation of cyclical surfaces from sinusoid, which is sine curve located on the surface of revolution. The parametric equations of a sinusoid and cyclical surface created on this sinusoid are in the paper.

K e y w o r d s. cyclical surface, sinusoid, surface of revolution, parametric expression of surface

1 INTRODUCTION

Sinusoid *s* is a planar curve determined by parametric equations:

$$x = u, y = \sin u, u \in \langle 0, 2\pi \rangle \qquad (1)$$

in Cartesian system of coordinates $(0, x, y)$ (Fig.1).

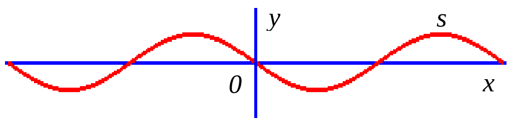

Fig.1: Sinusoid in E^2

Let sinusoid s' be a transformed curve *s* on the surface of revolution, so that coordinate axis *x* will be circle *c* with radius *r* and axis *y* will be meridian *m* of this surface, which is in Cartesian system of coordinates $(0, x, y, z)$ determined by parametric equations:

$$x = x(u,v), y = y(u,v), z = z(u,v), \qquad (2)$$

where $u \in \langle 0, 2\pi \rangle$ is parameter of circle and *v* is parameter of meridian of surface of revolution. Size of radius *r* of the parallel circle *c* determines number of periods of sinusoid s'. Parametric equations of sinusoid s' are:

$$x' = x'(u), y' = y'(u), z' = z'(u), u \in \langle 0, 2\pi \rangle. \qquad (3)$$

Cyclical sinusoidal surface Φ is created by the motion of a circle $k_0 = (P, r_0)$ alongside a sinusoid s' whereas its center is point lying on the curve $P \in s'$ and the circle lies in its normal plane. Then, the parametric equations of cyclical surface are:

$$x'' = x' + r_0 \cos u \, n_1 + r_0 \sin u \, b_1 \qquad (4)$$

$$y'' = y' + r_0 \cos u \, n_2 + r_0 \sin u \, b_2 \qquad (5)$$

$$z'' = z' + r_0 \cos u \, n_3 + r_0 \sin u \, b_3 \qquad (6)$$

where $\mathbf{t}(t_1, t_2, t_3), \mathbf{n}(n_1, n_2, n_3), \mathbf{b}(b_1, b_2, b_3)$ are the unit vectors of tangent, basic normal and binormal of its trihedron (P, t, n, b) [1]

$$\mathbf{t}(u) = (t_1, t_2, t_3) = \frac{\mathbf{r}'(u)}{|\mathbf{r}'(u)|} \qquad (7)$$

$$\mathbf{b}(u) = (b_1, b_2, b_3) = \frac{\mathbf{r}'(u) \times \mathbf{r}''(u)}{|\mathbf{r}'(u) \times \mathbf{r}''(u)|} \qquad (8)$$

$$\mathbf{n}(u) = (n_1, n_2, n_3) = \mathbf{b}(u) \times \mathbf{t}(u) \qquad (9)$$

where vector function of the sinusoid s' is

$$\mathbf{r}(u) = (x'(u), y'(u), z'(u)) \qquad (10)$$

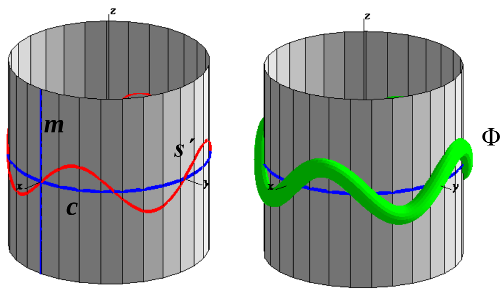

Fig. 2: Sinusoid s' and surface Φ on a cylinder

In Fig.2 is displayed sinusoid s' on the cylindrical surface with parametric equations:

$$x = r \cos u, y = r \sin u, z = v, \; v \in \langle -3, 3 \rangle \qquad (11)$$

$$x' = r \cos u, y' = r \sin u, z' = \sin v', v' = u \, r \qquad (12)$$

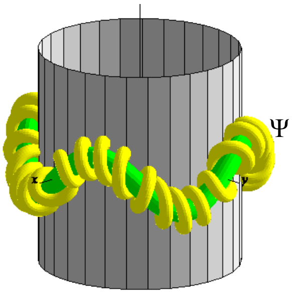

Fig. 3: Cylindrical helical surface Ψ

Fig.3 displays a cylindrical helical surface Ψ created by the helical motion of a circle whereas axes of helical movement is tangent *t* of the sinusoid *s*´.

2 DISPLAYING SINUSOID AND CYCLICAL SINUSOIDAL SURFACES ON SURFACES OF REVOLUTION

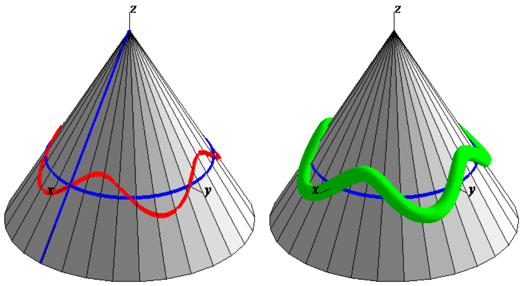

Fig. 4: Sinusoid *s*´and surface Φ on a cone

Fig.4 displaying sinusoid *s*´ on the conical surface with parametric equations:

$$x = r\ v\cos u,\ y = r\ v\sin u,\ z = h(1-v) \qquad (13)$$

$$x' = r'\cos u,\ y' = r'\sin u,\ z' = h\ \sin(r\,u)/d \qquad (14)$$

where

$$d = \sqrt{r^2 + h^2},\ r' = r(d - \sin(r\,u))/d \qquad (15)$$

and parameter $v \in \langle 0{,}1.5 \rangle$, *h* is height of a cone and *r* is its radius.

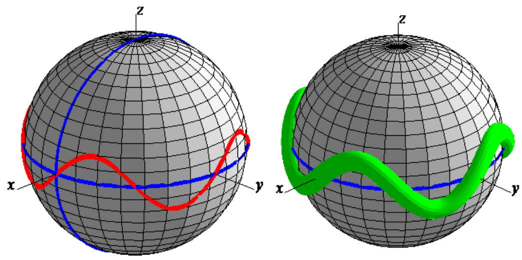

Fig. 5: Sinusoid *s*´ and surface Φ on a sphere

Fig.5 illustrating sinusoid *s*´ on the spherical surface with parametric equations:

$$x = r\cos u\cos v,\ y = r\sin u\ \cos v,\ z = r\sin v \qquad (16)$$

$$x' = r\cos u\cos v',\ y' = r\sin u\ \cos v',\ z' = r\ \sin(r\,u) \qquad (17)$$

where parameters $v \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$, $v' = \sin(r\ u)/r$ and *r* is radius of the spherical surface.

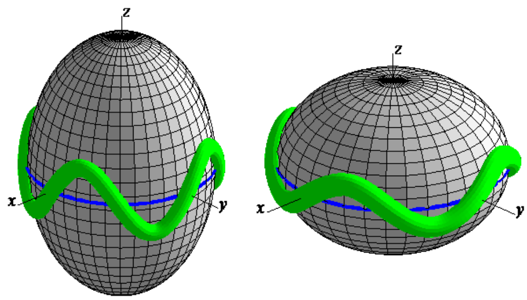

Fig. 6: Sinusoidal surfaces Φ on ellipsoids

Fig.6 displaying sinusoidal surfaces created on the ellipsoidal surfaces (prolate and oblate) with parametric equations:

$$x = a\cos u\cos v,\ y = a\sin u\ \cos v,\ z = b\sin v \qquad (18)$$

$$x' = a\ \cos u\ \cos v',\ y' = a\ \sin u\ \cos v',\ z' = b\sin v' \qquad (19)$$

$$v' = 2\sin(a\,u)/\left[1.5(a+b) - \sqrt{ab}\right] \qquad (20)$$

where parameter $v \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$, and *a,b* are semimajor and semiminor axes of the ellipsoidal surfaces, $a < b$ in first case and $a > b$ in second case.

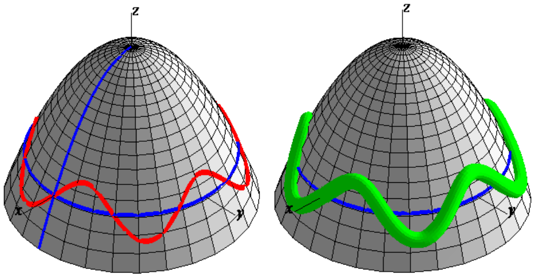

Fig. 7: Sinusoid *s´* and surface Φ on a paraboloid

Fig.7 displaying sinusoid s' on a paraboloidal surface with parametric equations:

$$x = v\cos u,\ y = v\sin u,\ z = h - v^2 / 2p \quad (21)$$

$$x' = v'\cos u,\ y' = v'\sin u,\ z' = h - v'^2 / 2p \quad (22)$$

$$v \in \left\langle 0, \sqrt{2ph} \right\rangle,\ v' = \sqrt{2ph} - \sin\left(u\sqrt{2pd}\right) / p \quad (23)$$

where h is height of paraboloid and p is its parameter.

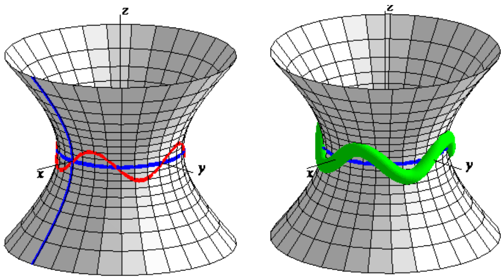

Fig. 8: Sinusoid *s´* and surface Φ on hyperboloid

Fig.8 displaying sinusoid s' and cyclical surface on a single sheet of hyperboloid with parametric equations:

$$x = a\cos u / \cos v,\ y = a\sin u / \cos v,\ z = b\,\mathrm{tg}v \quad (24)$$

$$x' = a\cos u / \cos v', y' = a\sin u / \cos v', z' = b\,\mathrm{tg}v' \quad (25)$$

$$v' = \sin(a\,u) / \left(\mathrm{tg}1\sqrt{a^2 + b^2}\right) \quad (26)$$

and a, b are semimajor and semiminor axes of the hyperboloidal surface and parameter $v \in \langle -1,1 \rangle$.

5 CONCLUSION

These curves and surfaces can be useful for technical engineers and designers in particular.

I wanted to demonstrate in this paper that mathematics, specially geometry is a powerful tool for technical engineer. If having any idea, at first, he shall take pencil and paper and put this idea on the paper. Then he cab describe this geometric problem mathematically. Then, using a suitable software he has to draws it on the computer. Students of our Faculty argue that mathematics and geometry in his practice is no longer needed, because the computer will do everything. But they are wrong, because no computer by itself is able to solve anything.

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