APPLICATION OF THE METHOD OF SMALL ALTERATIONS TO ANALYSE THE FUNDAMENTAL PARAMETERS OF A SMALL TURBOJET ENGINE MPM-20

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The presented article describes a mathematical method which analyses the relationship of the thermodynamic parameters of each flow section of a turbojet engine. By using the relations for the jet engine thermodynamic cycle it is possible to determine equations for computing small alterations. These equations can be defined as linear functions that describe particular thermodynamic processes. The method can be used for various purposes e.g. the engine design or diagnostic of engine technical condition

K e y w o r d s: jet engine, thermodynamic cycle, small alterations

1 INTRODUCTION

For purpose of design, as well as for further investigation of turbojet engines are currently used many sophisticated methods for computing the thermodynamic processes with a sufficiently high precision. In practise, however there are many problems that can´t be solved by the conventional thermodynamic relations. These are for example complexities of mutual influencing of each variables of engine sections as compressor, combustion chamber, gas turbine and exhaust jet nozzle and their impact to the engine characteristics. Examination of the mutual influences of the engine parameters while changing the fuel flow Q_p , or the exhaust nozzle diameter A_5 is very difficult. This stems from the need of knowledge of the engine dynamic processes that are set for each engine and are given by its size and performance characteristics. Currently the close attention is paid to the modelling of aircraft engines because the result of modelling the mathematical model of an aircraft engine is utilized for various purposes from designing up to diagnostics of aircraft engine technical condition.[8] This method is the most accurate of all the analytical methods, though with relatively difficult calculus. For investigation of the impact of basic parameters to engine characteristics can be also the method of small alterations used. This article can be used as a good source of information about the method of small alterations used for investigation of the mutual influence of the turbojet engine thermodynamic parameters.

2 THE METHOD OF SMALL ALTERATIONS

The method of small alterations is applicable only for relatively small changes of parameter values, as follows from its title. Disadvantage of this method is relatively small field of utilization because of increasing the deviation of real function from a linear substitute and applicability for higher engine-operating modes only. The engine thermodynamic variables are in close interrelations and changing the value of one can affect the alterations of the rests. The aim of the research is therefore to determine the impact of any thermodynamic parameter alteration, as well as the impact of engine technical condition to the engine performance.

To clarify the method of a small alteration assume, that there is a functional dependency

$$y = f(x)$$

which for value x = a reaches the value y = b. Matter is, which values reach the function y, how much will affect the alteration of x by Δx. Using the Taylor series we get increase of function y.

$$\Delta y = f'(a)\Delta x + \frac{1}{2!} f''(a)(\Delta x)^2 + \frac{1}{3!} f'''(a)(\Delta x)^3 ... \qquad (1)$$

Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. [9] The infinite sum permits to determine a function gain Δy. The method of a small alterations is based on the assumption, that Δx is relatively small, so its powers higher than 1 can be declared as negligible. In this case we can rewrite the formula 1 to form:

$$\Delta y = f'(a)\Delta x \qquad (2)$$

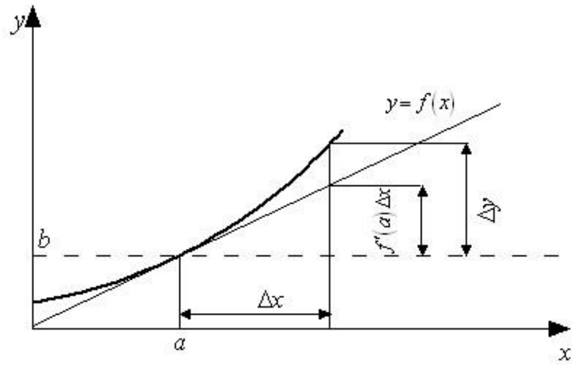

Fig.1 Linearized function

The difference between the values of Δy determined by equations (1) and (2) will be as small as Δx is, and more as the real function is closer to linear substitution.

In case, that there is a function z = f1 (x, y, t, u, v) and y = f2 (x, t) currently, we can define them as:

$$\begin{aligned}\Delta z &= \frac{\partial f_1}{\partial x}\cdot\Delta x + \frac{\partial f_1}{\partial y}\cdot\Delta y + \frac{\partial f_1}{\partial t}\cdot\Delta t + \frac{\partial f_1}{\partial u}\cdot\Delta u + \frac{\partial f_1}{\partial v}\cdot\Delta v \\ \Delta y &= \frac{\partial f_2}{\partial x}\cdot\Delta x + \frac{\partial f_2}{\partial t}\cdot\Delta t\end{aligned} \tag{3}$$

Where:

$$\frac{\partial f_1}{\partial x}=C_1, \frac{\partial f_1}{\partial y}=C_2, \frac{\partial f_1}{\partial t}=C_3, \frac{\partial f_1}{\partial u}=C_4, \frac{\partial f_1}{\partial v}=C_5$$

$$\frac{\partial f_2}{\partial x}=C_6, \frac{\partial f_2}{\partial t}=C_7$$

C_1 to C_7 are numerical coefficients that correspond to partial derivatives at the point, where the alteration begins. For example x = x0, etc. The method of small alterations is thus based on linearization of functions around the point where the change starts from.

3 LINEARIZATION OF THE ENGINE THERMODYNAMIC EQUATIONS FOR SMALL ALTERATIONS

This part contains a detailed description of some basic equations of jet engine thermodynamic cycle, and consequent derivation of equations for small alterations of basic engine parameters.

3.1 Air compression in compressor

The work needed to compress 1 kg of air is defined by formula:

$$W_{KC} = \frac{\kappa}{\kappa-1}\cdot RT_{1C}\left[\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1\right]\cdot\frac{1}{\eta_{KC}} \tag{4}$$

Where:

κ – Poisson´s constant (isentropic exponent)

R – gas constant

T_{1C} – temperature in compressor inlet

π_{KC} – overall pressure ratio

η_{KC} – compressor efficiency

Differential of previous formula is:

$$dW_{KC} = \frac{\partial W_{KC}}{\partial T_{1C}}dT_{1C} + \frac{\partial W_{KC}}{\partial \pi_{KC}}d\pi_{KC} + \frac{\partial W_{KC}}{\partial \eta_{KC}}d\eta_{KC}$$

If κ and R are constants, than formula (4) can be adjusted as:

$$dW_{KC} = \frac{\kappa}{\kappa-1}R\left[\frac{\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1}{\eta_{KC}}dT_{1C} + \frac{T_{1C}}{\eta_{KC}}\frac{\kappa-1}{\kappa}\pi_{KC}^{-\frac{1}{\kappa}}\cdot d\pi_{KC} - \right.$$
$$\left. -\frac{T_{1C}}{\eta_{KC}^2}\left(\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1\right)d\eta_{KC}\right]$$

By replacing the differential by difference can be written:

$$\Delta W_{KC} = \frac{\kappa}{\kappa-1}R\left[\frac{\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1}{\eta_{KC}}\Delta T_{1C} + \frac{T_{1C}}{\eta_{KC}}\frac{\kappa-1}{\kappa}\pi_{KC}^{-\frac{1}{\kappa}}\cdot \Delta\pi_{KC} - \right.$$
$$\left. -\frac{T_{1C}}{\eta_{KC}^2}\left(\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1\right)\Delta\eta_{KC}\right]$$

The formula determines the affect of alteration ΔW_{KC} to other variables. To determine its relative change, logarithmic calculation is needed:

$$\ln W_{KC} = \ln\left(\frac{\kappa}{\kappa-1}R\right) + \ln T_{1C} + \ln\left(\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1\right) - \ln\eta_{KC}$$

First derivative of logarithm can be written:

$$\frac{dW_{KC}}{W_{KC}} = \frac{dT_{1C}}{T_{1C}} + \frac{\frac{\kappa-1}{\kappa}\pi_{KC}^{\frac{\kappa-1}{\kappa}}}{\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1}\frac{d\pi_{KC}}{\pi_{KC}} - \frac{d\eta_{KC}}{\eta_{KC}}$$

And by replacing the differential with difference we get:

$$\partial W_{KC} = \partial T_{1C} + \frac{\frac{\kappa-1}{\kappa}\pi_{KC}^{\frac{\kappa-1}{\kappa}}}{\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1}\cdot\partial\pi_{KC} - \partial\eta_{KC} \tag{5}$$

Where: $\dfrac{\frac{\kappa-1}{\kappa}\pi_{KC}^{\frac{\kappa-1}{\kappa}}}{\pi_{KC}^{\frac{\kappa-1}{\kappa}}-1} = K_1$

K_1 is the impact ratio of variable π_{KC} to W_{KC}, and its value will be constant for each default values of π_{KC}.

It is clear from equation (5) that increasing the temperature on compressor inlet T_{1C} by 1% affects the same increase of total compression work W_{KC}, and decreasing of the efficiency of compressor $\partial\eta_{KC}$ by 1% affects decreasing of the W_{KC} by 1%.

The affect of changing the overall pressure ratio π_{KC} by 1% to W_{KC} depend on the initial value of π_{KC}, because $K_1 = f(\pi_{KC})$.

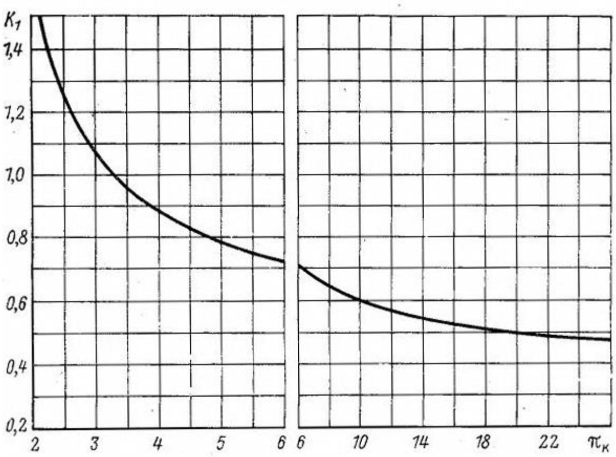

Fig.2 Dependency of coefficient K_1 on overal pressure ratio

The increase of compressed air temperature T_{2C} can by defined from formula:

$$T_{2C} - T_{1C} = \frac{\kappa - 1}{\kappa R} \cdot W_{KC}$$

$$\ln\left(T_{2C} - T_{1C}\right) = \ln\frac{\kappa - 1}{\kappa R} + \ln W_{KC}$$

Derivative of previous formula and its replacement with difference can be written as:

$$\partial\left(T_{2C} - T_{1C}\right) = \partial W_{KC} \tag{6}$$

The total temperature of compressed air is :

$$T_{2C} = T_{1C} + (T_{2C} - T_{1C})$$

$$\ln T_{2C} = \ln\left[T_{1C} + \left(T_{2C} - T_{1C}\right)\right]$$

$$\partial T_{2C} = \frac{T_{1C}}{T_{1C} + \left(T_{2C} - T_{1C}\right)} \cdot \partial T_{1C} + \frac{T_{2C} - T_{1C}}{T_{1C} + \left(T_{2C} - T_{1C}\right)} \cdot \partial\left(T_{2C} - T_{1C}\right) \tag{7}$$

Impact factor K_2 will be:

$$K_2 = \frac{T_{2C} - T_{1C}}{T_{1C} + \left(T_{2C} - T_{1C}\right)} = \frac{1}{1 + \frac{\eta_{KC}}{\pi_{KC}^{\frac{\kappa-1}{\kappa}} - 1}}$$

and therefore $\frac{T_{1C}}{T_{1C} + \left(T_{2C} - T_{1C}\right)} = 1 - K_2$

Consequently, the equation (7) can be written to form:

$$\partial T_{2C} = \partial T_{1C} + K_1 K_2 \partial \pi_{KC} - K_2 \partial \eta_{KC} \tag{8}$$

3.2 Heating the air in the combustion chamber

The equation comes out from the mass flow rate of fuel supplied to the main combustion chamber Q_{pal}, where assuming that mass flow of air and gas are of the same value, only one heat capacity ratio cp_{str} is used:

$$Q_{pal} = \frac{Q.cp_{str}\left(T_{3C} - T_{2C}\right)}{H_u \eta_{sp}}$$

By applying this equation to the small alterations can be written:

$$\partial Q_{pal} = \partial Q - \partial \eta_{SP} + \frac{T_{3C}}{T_{3C} - T_{2C}} \partial T_{3C} - \frac{T_{2C}}{T_{3C} - T_{2C}} \partial T_{2C}$$

if $\frac{T_{3C}}{T_{3C} - T_{2C}} = K_3$, then

$$\partial Q_{pal} = \partial Q - \partial \eta_{SP} + K_3 \partial T_{3C} - (K_3 - 1) \partial T_{2C} \tag{9}$$

3.3 Expansion of the gas in gas turbine

Similarly as for compressor, the total work is needed.

$$W_{TC} = \frac{\kappa'}{\kappa' - 1} R' T_{3C} \left[1 - \frac{1}{\pi_{TC}^{\frac{\kappa'-1}{\kappa'}}}\right] \eta_{TC}$$

Assuming, that κ′ and R′ are constants, after implementation for small alterations the total work is:

$$\partial W_{TC} = \partial T_{3C} + \partial \eta_{TC} + K_4 \partial \pi_{TC} \tag{10}$$

Where $K_4 = \frac{\frac{\kappa' - 1}{\kappa'}}{\left(\pi_{TC}^{\frac{\kappa'-1}{\kappa'}} - 1\right) \pi_{TC}}$

K_4 is the impact factor of total turbine pressure ratio π_{TC} to the total turbine work.

Differential of the total gas temperature T_{4C} can be obtained from formula:

$$T_{4C} = T_{3C} - \left(T_{3C} - T_{4C}\right)$$

Using the principle described in 3.1 a final shape of equation for gas temperature T_{4C} alteration is:

$$\partial T_{4C} = \frac{T_{3C}}{T_{3C} - \left(T_{3C} - T_{4C}\right)} \partial T_{3C} - \frac{T_{3C} - T_{4C}}{T_{3C} - \left(T_{3C} - T_{4C}\right)} \partial\left(T_{3C} - T_{4C}\right)$$

Since the alteration of temperatures T_{3C} and T_{4C} is equal to the alteration of turbine work, previous formula can be rewritten to form:

$$\partial\left(T_{3C} - T_{4C}\right) = \partial T_{3C} + \partial \eta_{TC} + K_4 \partial \pi_{TC}$$

By modification of this formula, the equation of temperature T_{4C} alteration is defined by next formula:

$$\partial T_{4C} = \partial T_{3C} - K_5 \partial \eta_{TC} - K_4 K_5 \partial \pi_{TC} \tag{11}$$

Where $K_5 = \frac{T_{3C} - T_{4C}}{T_{3C} - \left(T_{3C} - T_{4C}\right)} = \frac{1}{\frac{1}{\eta_{TC}\left(1 - \pi_{TC}^{1-\kappa'} - 1\right)}}$

The following table shows the relations needed to compute the impact factors of some thermodynamic parameters to parameters of engine performance. Relations are defined by using coefficients K_i.

Tab.1[2]

	$\delta\pi_{KC}$	δT_{3c}
δF	$\frac{1}{2}K_9\left[\frac{2\left(K_7 - 1\right)}{K4}\right]\left(K_4 - K_1\right) - K_1 K_5$	$\frac{1}{2}K_9\left[\frac{2\left(K_7 - 1\right)}{K4}\right] + K_5 + 1$
δcm	$-\frac{1}{2}K_9\left[\frac{2\left(K_7 - 1\right)}{K_4}\left(K_4 - K_1\right) - - K_1 K_5\right] - K_1 K_2\left(K_3 - 1\right)$	$-\frac{1}{2}K_9\left(\frac{2\left(K_7 - 1\right)}{K_4} + K_5 + 1\right) + K_3$

	$\delta\eta_{KC}$	$\delta\eta_{TC}$
δF	$\frac{1}{2}K_9\left[\frac{2(K_7-1)}{K4}+K_5\right]$	$\frac{1}{2}K_9\left[\frac{2(K_7-1)}{K4}\right]$
δcm	$-\frac{1}{2}K_9\left(\frac{2(K_7-1)}{K4}+K_5\right)+$ $+K_2(K_3-1)$	$-\frac{1}{2}K_9\left[\frac{2(K_7-1)}{K4}\right]$

	$\delta\sigma_{SK}$, $\delta\sigma_{VS}$, $\delta\sigma_{VD}$	δQ	$\delta\eta_{SP}$
δF	$\frac{1}{2}K_9 2(K_7-1)$	1	0
δcm	$\frac{1}{2}K_9 2(K_7-1)$	0	-1

σ_{SK}- pressure recovery in combustion chamber
σ_{VS}- intake pressure recovery
σ_{VD}- pressure recovery in exhaust system
Q- mass flow of air

Where coefficients from K_7 till K_9 are defined by equation of mass flow of gas in the exhaust nozzle. Their description and extrapolation is due to this contribution range omitted.

Complete analysis of the equation of mass flow of gas thru exhaust nozzle as well as other coefficients of impact can be found in source [4] (Čerkez, A., Ja.).

4 APPLICATION OF THE METHOD OF SMALL ALTERATIONS TO DIAGNOSTIC THE JET ENGINE TECHNICAL CONDITION

Based on the analysis of all thermodynamic relations similarly to a part 3 it is possible to determine the impact of any variables on the others. Alteration of efficacy parameter of each engine part can be caused by mechanical wastage, leakage and partial damage of engine part.

The following table shows the impact coefficients calculated for a small turbojet MPM-20 at maximum operating mode. Computed table of impacts can be used to diagnose the mechanical wastage of compressor, gas turbine and combustion chamber.

Tab.2 [4]

	$\delta\sigma_{vst}$	$\delta\sigma_{sk}$	$\delta\sigma_{tr}$	$\delta\eta_{kc}$	$\delta\eta_{tc}$	δA_{rk}	δA_5	δQ
$\delta\pi_{tc}$	0,584	0,507	0,993	-0,039	-0,006	-0,486	0,409	-0,205
$\delta\pi_{kc}$	-0,045	-0,171	-0,076	-0,063	-0,066	-0,095	-0,031	0,016
$\delta\pi_{tr}$	0,371	0,322	0,642	-0,025	-0,060	0,391	-0,440	-0,220
δQ	1,277	1,061	0,471	0,392	0,407	0,590	0,194	-0,097
δT_{2c}	0,012	0,045	0,020	-0,357	0,017	0,025	0,008	-0,004
δT_{3c}	-0,643	-0,464	-1,094	-0,910	-0,946	0,439	-0,451	0,225
δF	1,715	1,488	-0,318	-0,113	-0,275	1,806	-1,033	1,017
δQ_{pal}	0,179	-0,140	-0,306	-1,162	-1,208	1,321	-0,575	1,750

Practical use of the calculated coefficients of influence can be demonstrated on simple example. If the intake pressure recovery decreases by $\delta\sigma_{vs}=-1\%$ and efficiency of compressor decreases at the same time by $\delta\eta_{KC}=-1\%$, then this alteration can be seen on engine thrust as:

$$\delta F = 1{,}715\cdot\delta\sigma_{vs} - 0{,}113\cdot\delta\eta_{KC} =$$
$$= 1{,}715\cdot(-1) - 0{,}113\cdot(-1) = \underline{\underline{-1{,}602}}$$

The result signifies the decrease of total engine thrust by 1,602%.

Due to Tab.2 can be concluded, that most significant impact to total thrust F has the intake pressure recovery $\delta\sigma_{vst}$ and the area of turbine stator δA_{rk}.

The fuel flow is significantly affected by air flow Q, area of turbine stator δA_{rk} and by rate of compressor and turbine efficiency.

One of the engine performance parameters is the specific fuel consumption cm, this can be influenced rapidly by changing the values of intake pressure recovery and pressure recovery of combustion chamber σ_{SK}.

Next table Tab.3 shows the impact of total compressor efficiency to the engine parameters (turbine expansion, compressor pressure ratio, expansion in jet nozzle, gas temperature and fuel consumption per hour) and demonstrates the accuracy by comparing the calculated and measured values.

Tab.3[3]

	$\delta\pi_{TC}$	$\delta\pi_{KC}$	$\delta\pi_{VD}$	δT_{3c}	δc_h
$\eta_{KC,1}$	2,1975	3,5327	1,540	1199,65	72,2154
$\eta_{KC,6}$	2,1982	3,5417	1,546	1214,17	72,22
$\eta_{KC,nam.}$	-0,03	-0,41	-0,392	-1,111	-0,12
$\eta_{KC,vyp.}$	-0,254	-0,4064	-0,381	-1,2207	-0,1164
Δ %	0,046	0,0036	0,0009	0,1097	0,0036

5 CONCLUSION

The article was aimed to explain the method, which is used to determine the response of engine performance parameters while changing one of engine thermodynamic parameter. To clarify the principle, description of basic thermodynamic relations for jet engine was needed. The method was verified by using the tables of coefficients on simple example, and also the accuracy of impact computing was presented. Deviations of measured values compared with computed may be caused by low quality of used sensors or non-steady temperature fields. The method of small alterations despite of its disadvantages described in section 2 is very useful for designers as well as for diagnostic the engine technical condition and for investigation of turbojet thermodynamic processes.

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